NOTE AND CORRESPONDENCE

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A Fractal Analysis of Earthquakes in West Taiwan

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ABSTRACT

The fractal dimension is calculated for earthquakes in west Taiwan. The correlation integral algorithm proposed by Hirata *et al.* (1987) is used to estimate the values of fractal dimensions for earthquakes in two parts, separated approximately by the latitude of 24°N in west Taiwan. Results show that the fractal dimesion value (1.304±0.002) for northern events is less than the value (1.599±0.002) for southern events. This indicates that northern events are more heterogeneous than southern events. Besides, the fractal dimension is negatively correlated with the b value.

Fractal is a general concept of self-similarity of natural phenomena (Mendelbrot, 1983). Seismicity is specified with self-similarity first from the Gutenberg-Richter's frequency-magnitude relation (Gutenberg and Richter, 1944). By using the two-point correlation function, Kagan and Knopoff (1980) showed that the spatial distributions of earthquakes have stochastic self-similarities and that the fractal dimension changes from 1 to 1.5 with the hypocentral depth for worldwide earthquakes. From a box-counting algorithm, Sadovskiy *et al.* (1984) displayed that the spatial distribution of earthquakes both on a worldwides scale and in the local Nurek-region, earthquakes is fractal. From a fracture experiment on Oshima granite, Hirata *et al.* (1987) found that the spatial distribution of hypocenters of acoustic emission is a fractal, and the fractal dimension is 2.75, 2.66, and 2.25 at the three stages of primary creep, secondary creep and tertiary creep, respectively. Hirata (1989) estimated the fractal dimension of the spatial distribution of earthquakes in the Tohoku region, Japan. Hirata and Imoto (1991) analyzed the multifractal dimensions of spatial distribution of microearthquakes in the Kanto region. Hirabayashi *et al.* (1992) analyzed the multifractal dimensions of earthquakes in California, USA, Japan, and Greece. They reported that the fractal dimensions are different in the three regions, and indicate different degrees of heterogeneity of earthquakes in the three regions: the most heterogeneous in Japan, medium in Greece and the least in California, USA.

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(Takayasu, 1990): similarity dimension D_S, capacity dimension D_{CA}, information dimension D_I, and correlation dimension D_C. In general, D_S=D_{CA}>D_I>D_C. The fractal dimension considered by Sadovskiy *et al.* (1984) is the capacity dimension. Theoretically, Aki (1981) studied the relation of the capacity dimension to the b value of Gutenberg-Richter's frequency-magnitude relation. The correlation dimension was used by Kagan and Knopoff (1980), Hirata *et al.* (1987) and Hirata (1987) in different forms to estimate the fractal dimension of seismicity.

The collision of the Philippine-Sea and Eurasian plates together with the spreading of the Okinawa trough have resulted in high seismicity in the Taiwan region (Tsai *et al.*, 1977). Since the hypocentral distribution of earthquakes is quite inhomogeneous in this region (Wang, 1988), the fractal dimension must be a significant indicator to show this inhomogeneity. The error of earthquake location is higher for offshore events than for inland events. Meanwhile, the seismicity in the Central Range is very low (Wang, 1988) and the inland seismic zone in east Taiwan is rather narrow. Hence, in this study, the correlation dimension defined by Hirata *et al.* (1987) is used for the evaluation of fractal dimension of hypocenters of earthquakes in west Taiwan. For simplification, the fractal dimension is hereinafter denoted as D.

The Taiwan Telemetered Seismographic Network has been installed since the end of 1972. A detailed description of this network can be found in Wang (1989). This network provides a good data base for earthquakes. However, before 1983 earthquake location was based on a velocity model deduced from a very limited data set and must be very uncertain. Since 1983, a velocity model inferred by Yeh and Tsai (1981) on the basis of a large number of observed data has been used for locating earthquakes, and the error of location should be reduced. The earthquakes that occurred during 1983-1986 are used in this study. The magnitude range of earthquakes is from 1 to 5. According to Wang (1988), most inland earthquakes in the region are located within the focal depth of 40 km. This focal depth is considered to be the value of the lower boundary of earthquakes selected. The maximum error of the determination of hypocenter for inland earthquakes is about 5 km. This value will lead to the lower bound of distance for the establishment of the correlation integral. The epicenters of earthquakes used are shown in Figure 1.

The correlation integrals C(r) for the hypocentral distributions (h_1, h_2, h_3, ..., h_N) were calculated with the following formula (Hirata *et al.*, 1987):

$$C(r) = 2N_r(R < r)/N(N-1) \tag{1}$$

where $N_r(R<r)$ is the number of pairs (h_i, h_j) with a distance smaller than r, and N is the number of events used. If the distribution has a fractal structure, C(r) is expressed by

$$C(r) \sim r^D, \tag{2}$$

where D is the correlation fractal dimension.

The existence of the Peikang basement high, first recognized by Stach (1957), in the area approximately between 23.5°N and 24.0°N, separates west Taiwan into two geological provinces (see Ho, 1982). From Figure 1, a small zone with low seismicity can be seen in such an area. Hence, the fractal dimension is estimated for earthquakes to the north and to the south of the Peikang basement high. The division is selected approximately along the latitude of 24°N and is denoted by a broken line in Figure 1. The number of event used is 643 for the northern part and 634 for the southern part. The correlation integral versus the

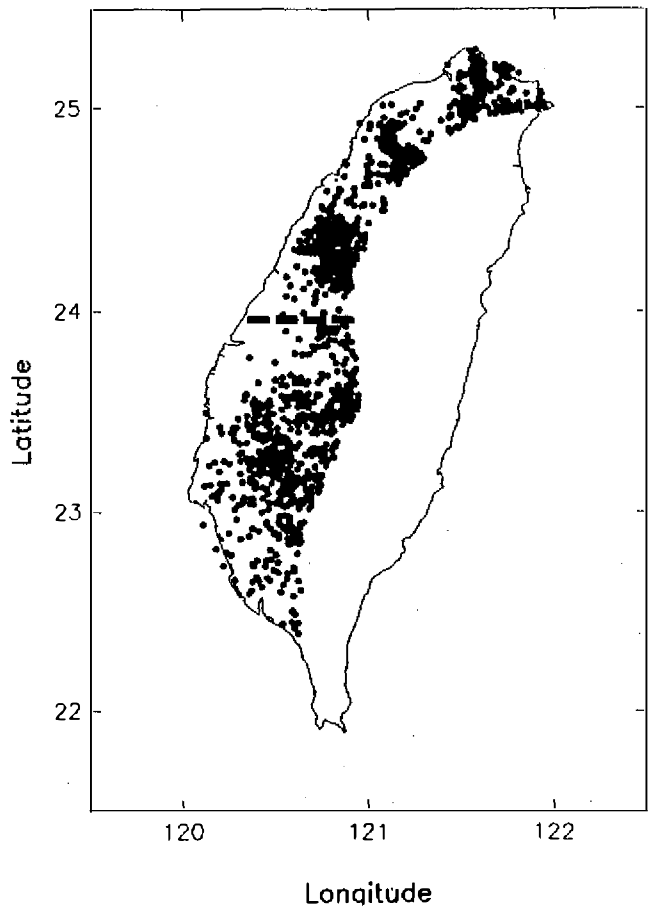

Fig. 1. The epicenters of earthquakes used in this study. The broken line near the latitude of 24°N divides the data set into two parts.

distance for the hypocentral distribution is plotted on a double natural logarithmic scale in Figure 2. The open circles and squares denote the data points for earthquakes in the northern and southern parts of west Taiwan, respectively. It can be seen that when the distance is less than a certain critical value r_c, the data points distribute almost along a straight line; while when the distance is greater than such a value, the two patterns of data points bend. Such a critical value is about 40 km, or $\ln(r_c)=3.7$, for southern events and about 25 km, or $\ln(r_c)=3.2$, for northern events. The r_c value for southern events is obviously greater than that for northern events. The bending of the pattern of data points indicates that the C(r) value for $r>r_c$ is less than the value estimated from the regression equation deduced from the data points with $r<r_c$. From Figure 1 it can be seen that the r_c value of northern events is almost equal to the smallest width value of the epicentral distribution of earthquakes, whereas the r_c value for southern events is almost equal to the maximum value of focal depth of earthquakes used, since the correlation integral algorithm is based on a circle in the two-dimensional space and a sphere in the three-dimensional space. For both the northern and southern parts of west

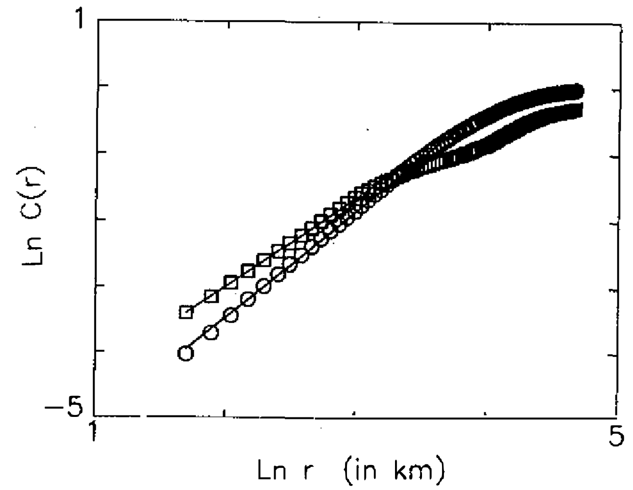

Fig. 2. Ln C(r) vs. Ln r: open squares for northern events and open circles for southern events. The solid lines represent the regression lines of the data points with r less than a certain value r_c: r_c=25 km (or ln r_c=3.2) for northern events and r_c=44 km (or ln r_c=3.8) for southern events.

Taiwan, the earthquakes used are located within a long box. As the r value is greater than the smallest edge of the box, the number of the pairs of events counted from the box must be less than the expected value based on a sphere with a radius of r. Therefore, the size of the smallest edge of the box would cause a so-called finite-size effect on the computed results. For the present situation, the critical size is the width of the box for northern events and the depth of the majority of earthquakes for southern events. The use of non-spherical distribution of earthquakes will limit the reliability of the results. In other words, the fractal dimension can be estimated only from the data points with $r<r_c$. For the data points with $r<r_c$, the slope values (i.e. the D values) inferred from the data points are 1.304±0.002 and 1.599±0.002 for northern and southern events, respectively. A non-integer value of fractal dimension represents the existence of voids in the object or set. From the viewpoint of earthquake occurrence, the existence of voids indicates the existence of small areas where the earthquake is not located. The smaller the value of fractal dimension, the bigger the number of voids or the higher the degree of heterogeneity of the object. Hence, the fact that the D value for northern events is smaller than the value for southern events displays a more heterogeneous distribution of earthquakes in the northern part of west Taiwan than in the southern part of west Taiwan. Figure 1 evidently shows a linear distribution of clusters of epicenters, which are separate in the north, but not so much in the south. In other words, Figure 1 displays a more heterogeneous distribution of earthquakes in northern part than in the southern part. However, the surface geology does not seem able to explain the difference of values of fractal dimension in the two parts of west Taiwan. The subsurface geology might be responsible for the distinction.

The b value of the Gutenberg-Richter relation and the fractal dimension D are two parameters to display the heterogeneity of materials where earthquakes occur. Wang (1988) calculated the b values of shallow earthquakes during 1973-1986 in Taiwan. In west Taiwan, his results show, on the average, a higher b value in the northern part (with a lower D value) than in the southern part (with a higher D value). For the present data set, the data points of logN vs. M for northern events (denoted by open squares) and southern events (denoted by open circles) are plotted in Figure 3. The solid lines represent the regression lines from the data points in the individual magnitude ranges: 2.5 to 3.5 for northern events and 2.5 to 4.0 for southern events. The b values are 1.778+0.040 and 0.923+0.020 for northern and southern events, respectively. Although the data used for estimating the b and D values are different, as shown in Figures 2 and 3, the result still shows a negative correlation between the b value and fractal dimension. Hirata (1989) also reported a negative correlation between the two parameters for earthquakes (with focal depth smaller than 60 km) in the so-called Tohoku region of Japan. However, Aki (1981) speculated that there is a positive relation between the two parameters in the form: D=3b/c, where c is the slope of log moment versus magnitude relation, and c is about 1.5. Hirata (1989) discussed this problem in details. He concluded that the fractal dimension of the geometry of fault planes used by Aki is the special case of the capacity dimension of asperity or barrier distribution in which all asperities or barriers are connected to each other without isolation, where dimension can be regarded as the fractal dimension of the surface of the fault plane. But, this is not necessarily true for observed seismicity, which is produced from various fault planes. Therefore, a negative correlation between the two parameters is reasonable.

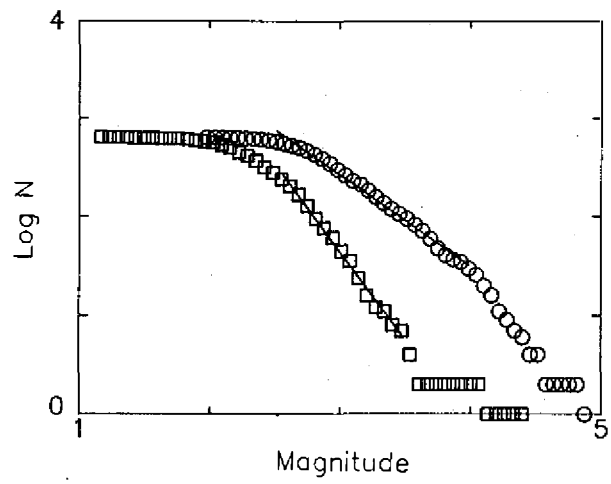

Fig. 3. Log N vs. M: open squares for northern events and open circles for southern events. The solid lines represent the regression lines of the data points in two magnitude ranges: 2.5 to 3.5 for northern events and 2.5 to 4.0 for southern events.

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