TAO, Vol.7, No.2, 159-166, June 1996

A New Method in Polarization P/S Filters of Vector Wavefield

CHIOU-FEN SHIEH[1]

(Manuscript received 1 July 1995, in final form 29 January 1996)

ABSTRACT

The characteristics of a vector wavefield can be investigated through the polarization method. This study presents an alternate polarization method to separate (or isolate) the longitudinal and the transverse waves in the elastic case. Besides, this method can be used to reduce nonlinear elliptical waves. First, the vector wavefield with vertical and the radial components is selected and the analytic signals are obtained through a Hilbert transform. Then, their polarization parameters, including phase difference, ellipticity and the strength of linear polarization, are calculated in the time domain. According to these parameters, the filter function of linear logitudinal and transverse waves are defined. Finally, the synthetic data are analyzed, thereby showing that it is very effective to remove unwanted wave types even when the S/N ratio is as low as 1.667.

(Key words: Polarization, P/S filters, Vector wavefield)

1. INTRODUCTION

The elastic wave, as one kind of vector wavefield, encompasses the linear motion of P- and S-waves, and the elliptical motion of Rayleigh waves as well. Most exploration geophysists are much interested in P-waves, or in S-waves after the development of shear wave exploration. Yet, most of the data obtained from the outfield include these waves, so the signals become unclear and even mixed up due to interferences. Many solutions, such as bandpass filter, velocity filter, ...etc., have been used to partially overcome this problem at least. The polarization filter (Flinn, 1965), resulting from the perspecting of particle motions has been employed in this field as one of the solutions. fAdditionally, the technique which reduces Rayleigh waves brought up by Shieh and Herrmann (1990) was considered a solution as well. In order to conduct a successful analysis, it is important to filter data so that it only contains one kind of wave (P- or S-). The main purpose of this study is to introduce a method to reach this goal.

Using the polarization method to analyze the characteristics of the elastic wave has already been widely discussed. Flinn (1965) first introduced the utilization of the covariance

[1] Institute of Seismology, National Chung Cheng University, Chiayi, Taiwan, R.O.C.

matrix in the real time domain to manipulate earthquake problems. Later, studies conducted by Samson and Olson (1981) a data-adaptive polarization filter in their design to enhance waves of specific polarization. Taner *et al.* (1979) and Rene *et al.* (1986) used analytic signals in the time domain for analysis, while Vidale (1986) employed the eigenvalue and eigenvector of the coherency matrix to determine the polarization parameters, which is considered to be more complete. Mott (1986) analyzed radar signals using the polarization method, and theoretically, although what he employed was still a analytic signal, it only involved 2-component data. The present study also deals with only 2-component data. Therefore, the data used here must be the signals received from the vertical and the radial components on the incident plane.

2. METHOD

Supposing the time series are recorded in the vertical (Z) and the radial (R) components, after the Hilbert transform, the signals can be expressed as:

$$z(t) = a_z(t)e^{-j\phi_z(t)}, r(t) = a_r(t)e^{-j\phi_r(t)}. \tag{1}$$

A time window is selected, and a coherency matrix is constructed:

$$C = \begin{pmatrix} < z(t)z^*(t) > & < z(t)r^*(t) > \\ < r(t)z^*(t) > & < r(t)r^*(t) > \end{pmatrix} = \begin{pmatrix} < a_z^2 > & < a_z a_r e^{j\phi} > \\ < a_z a_r e^{-j\phi} > & < a_r^2 > \end{pmatrix}, \tag{2}$$

where ϕ=ϕ_z-ϕ_r as the phase difference between the vertical and the radial components, the symbol $*$ represents the complex conjugate and $<>$ represents the average value in the time window. Matrix C is a non-negative Hermitian containing all the information needed to characterize the polarization parameters. Equation (2) was used by Vidale (1986) where he employed 3-component data in his study, but here, only 2-component data is used as Shieh and Herrmann did (1990). Equation (2) processed with eigenanalysis can be expressed as:

$$CU = \lambda U, \tag{3}$$

where λ is a 2 $\times$ 2 diagonal matrix, representing eignevalues, and U is a 2 $\times$ 2 eigenvector matrix. It is found that the eigenvector corresponding to the maximum eigenvalue takes the following form

$$U = \begin{pmatrix} z_r \\ r_r + jr_i \end{pmatrix} = \begin{pmatrix} \hat{a_z} \\ \hat{a_r}cos\phi - j\hat{a_r}sin\phi \end{pmatrix}, \tag{4}$$

where $\hat{a_z}$ and $\hat{a_r}$ are the normalized directional projections.

It is clear that the phase difference (ϕ) between the vertical and the radial components can be calculated by

$$\phi = tan^{-1}(\frac{-r_i}{r_r}). \tag{5}$$

For the pure longitudinal signal (P-wave), ϕ = 0°, and for the pure transverse signal (S-wave) ϕ = 180°. As for other nonlinear waves, this angle is located between 0° and 180°. From this perspective, two functions for passing P- and S-waves may be defined as:

$$P_c(t) = \frac{(1 + cos\phi)}{2}, S_c(t) = \frac{(1 - cos\phi)}{2}. \tag{6}$$

If the signal is pure P-wave, ϕ = 0, so $P_c(t)$ = 1, and $S_c(t)$ = 0. If the signal is pure S-wave, ϕ = 180, so $P_c(t)$ = 0, and $S_c(t)$ = 1. For other nonlinear waves, $P_c(t)$ and $S_c(t)$ are both smaller than 1, so Equation (6) can be used to suppress the nonlinear waves. Note that for the pure incident S-wave with an angle of incidence beyond the critical angle, the particle motion becomes elliptically polarized. Equation (6) can't isolate this kind of S-wave successfully unless the ellipticity is very small.

As for defining the strength of linear polarization (Vidale, 1986), it is stated as:

$$P_L = 1 - \frac{\lambda_2}{\lambda_1 + \lambda_2}, \tag{7}$$

where λ_1 and λ_2 are the maximum and minimum eigenvalues, respectively. The main feature of this formula is to suppress the noise. When noise does not exist, $\lambda_2 = 0$ and $P_L = 1$; on the other hand, $\lambda_2 \neq 0$, and $P_L < 1$ when noise exists.

In the final step, the nonlinear wave is reduced by ellipticity (e), and the function is defined as:

$$P_e = 1 - e \tag{8}$$

Ellipticity can be obtained by the rotational angle as proposed by Vidale (1986). Under the 2-component condition, the method is simplified by finding an angle with the exponential term, $cs = exp(j\theta)$, then multiplying the eigenvector (Equation 4) corresponding to the maximum eigenvalue and finally minimizing the real part by:

$$w = \sqrt{real(U(1,1) * cs)^2 + real(U(2,1) * cs)^2}. \tag{9}$$

The ellipticity is determined by:

$$e = \frac{\sqrt{1-w}}{w}. \tag{10}$$

The filter function of passing longitudinal wave may be defined by combining Equations (6), (7) and (8):

$$P(t) = P_c^2(t)P_L^2(t)P_e^4(t). \tag{11}$$

The filter function of passing the transverse wave is defined as:

$$S(t) = S_c^2(t)P_L^2(t)P_e^4(t). \tag{12}$$

3. FREQUENCY AND TIME DECOMPOSITION

In practical applications, two procedures need to be followed before the polarization method can be used; they are frequency and time decomposition. One particular reason is that wave types with different frequencies may arrive at the same time, such that the polarization properties are mixed up. Jurkevics (1988) developed a rigorous algorithm to solve this problem. Subsequently, Shieh and Herrmann (1990) slightly modified the cosine window into the cosine square window. The following is a summary of the algorithm.

The first step is to narrow-bandpass the input data over the entire frequency range (frequency decomposition). Second, a 50% overlap cosine square moving window is applied to the bandpassed data (time decomposition). The time-domain-moving window has a window width equal to $2/f_c$, where f_c is the central frequency of the bandpass filter. Third, the polarization filter function (Equation (11) or (12)) is computed in the tapered time series. Fourth, this function is multiplied to the entire tapered time series. Fifth, the steps from the second through the fourth are repeated until the entire time series is covered (the cosine square window effect is added). Finally, the steps from the first through the fifth are repeated until the entire frequency range is covered (the bandpass effect is added).

The purpose of decomposing the time series as a function of both frequency and time is to separate the contribution of signals with different arrival times and frequency responses. Obviously, simultaneous multiple arrivals with an identical frequency response can't be separated with such a method.

4. SYNTHETIC TEST

To start the synthetic test, five Ricker Wavelets with vertical and the radial (Z and R) components are simulated and listed in Figures 1-(a) and 1-(b), respectively. The first one is a pure longitudinal wave, while the fifth is a pure transverse wave. Others between the first and the fifth are nonlinear waves with phase differences (ϕ) of 45°, 90° and 135° (Shieh and Herrmann, 1990). Their particle motion can be found from the hodogram in Figure 1-(c). Figures 2-(a) and 2-(b) illustrate the signals being processed through the polarization filter (as Equations (11) and (12)). It is found that, in Figure 2-(a), only the longitudinal P-wave passes over, while the transverse S-wave and the nonlinear waves are all reduced or suppress to a lower level. Likewise, in Figure 2-(b), only the S-wave passes over. The filtered waveforms are slightly distorted at the beginning and the end due to the imperfection of bandpass filtering and frequency decomposition.

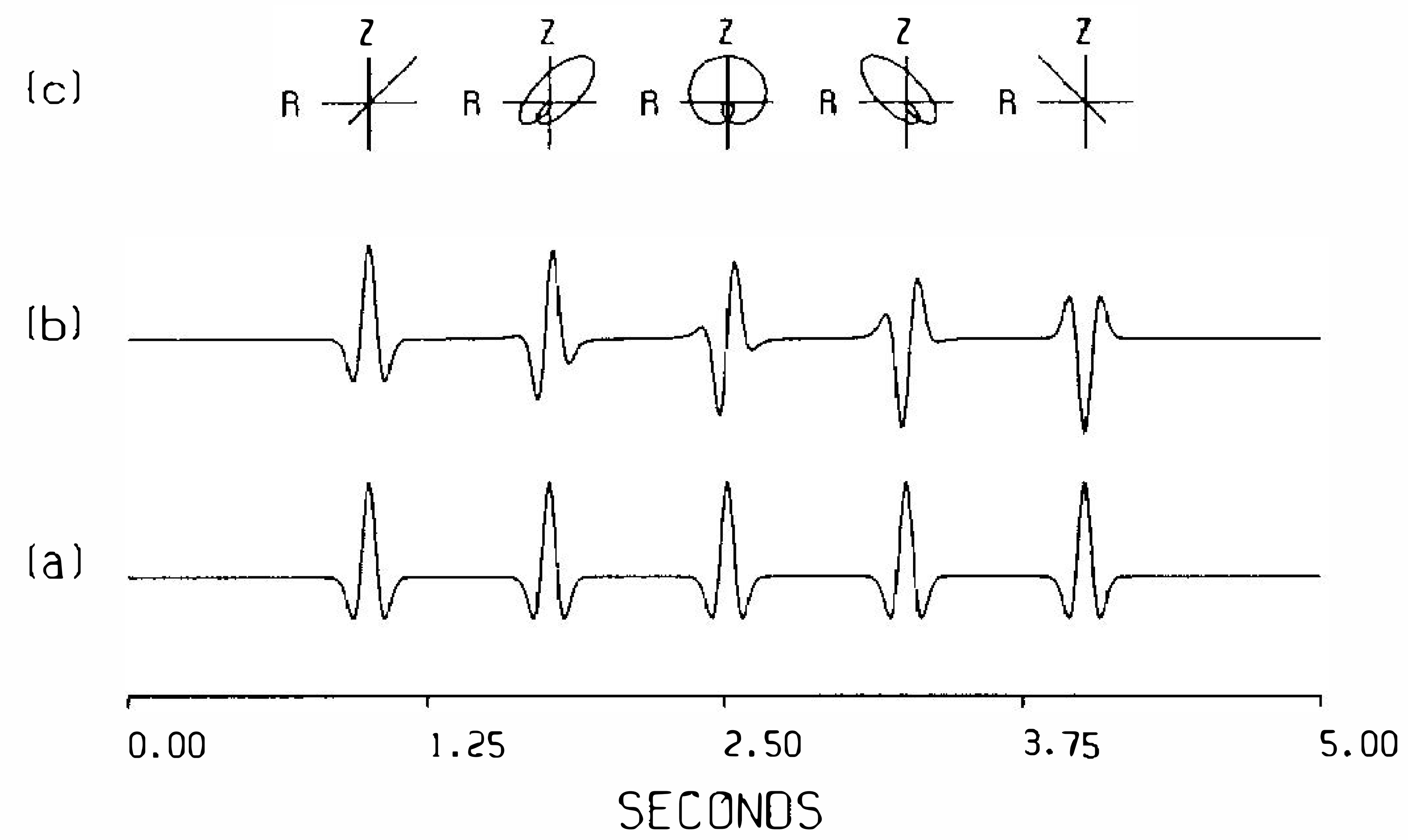

Fig. 1. Five 2-component Ricker wavelets with phase diferrence between (a) the vertical and (b) the radial components, 0°, 45°, 90°, 135° and 180° from the first to the fifth respectively. The wave types can be identified from the particle motion of (c), where the first and the fifth show linear motion while the other three show nonlinear motion.

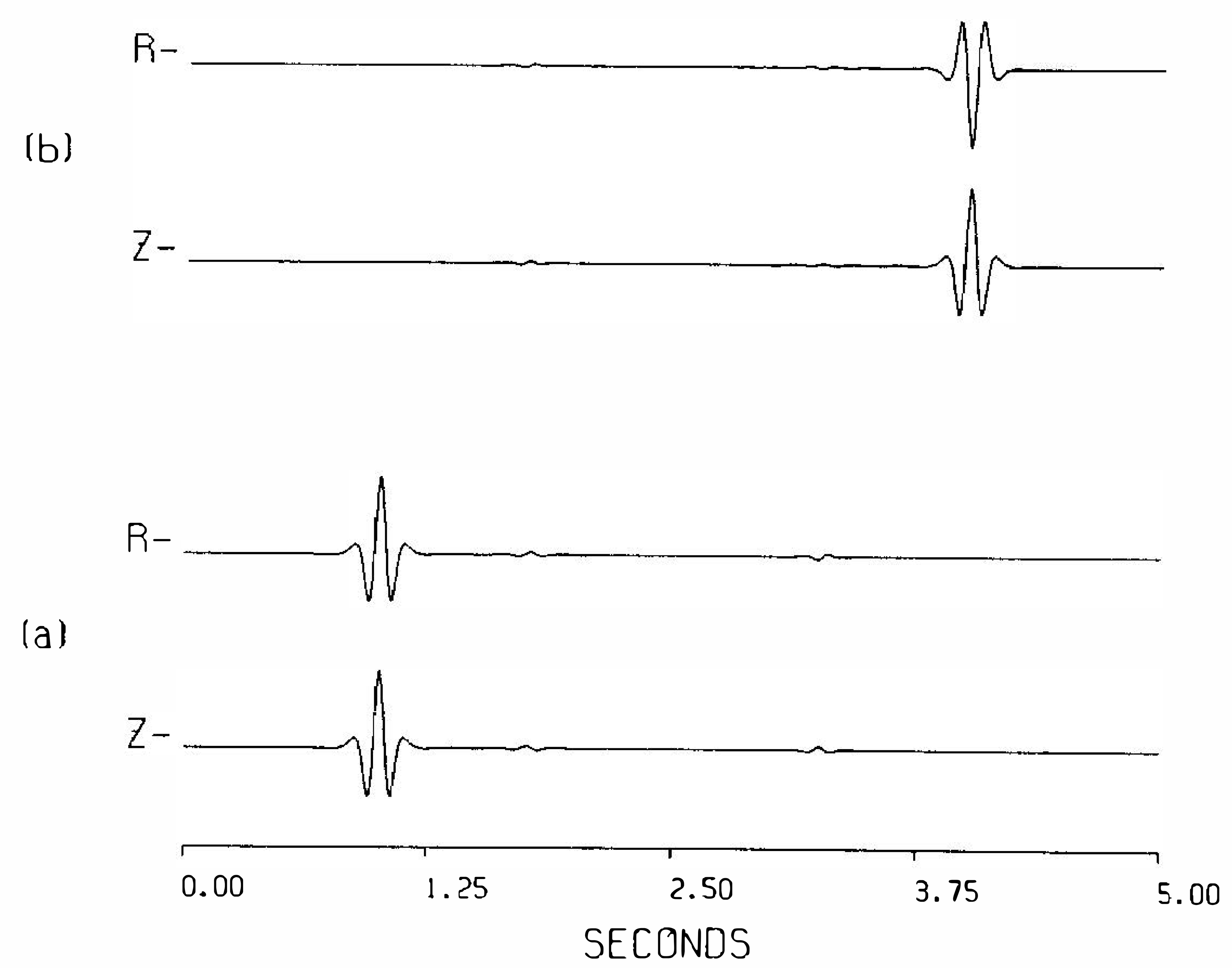

Fig. 2. Results of passing the polarization filter, (a) filtered P-waves using Equation (11), (b) filtered S-waves using Equation (12).

When noise exists, one may wonder if the function of the filter still works. To check its effectiveness, different levels of noise (from 10% to 60%) may be added to the original data in Figures 3-(a) and 3-(b). Then they can be made to pass over the polarization filter. Figures 4 and 5 show the results of passing P-waves and passing S- waves respectively. Although it is understood that the function of the filter decreases as the noise becomes larger, at the level of S/N =1.667 (60% noise), the function of the filter is still considered effective and acceptable in pratice.

5. CONCLUSION

It is very effective to use the polarization method to study the vector wavefield. Based on this viewpoint, this study presents a new filtering method. In addition to its function of reducing noise and the nonlinear waves, this method can be used to isolate linear longitudinal and transverse waves. Furthermore, it can also be applied to seismic waves, underwater acoustics, radar signals, ...etc., if the data are recorded by the 2-component receiver.

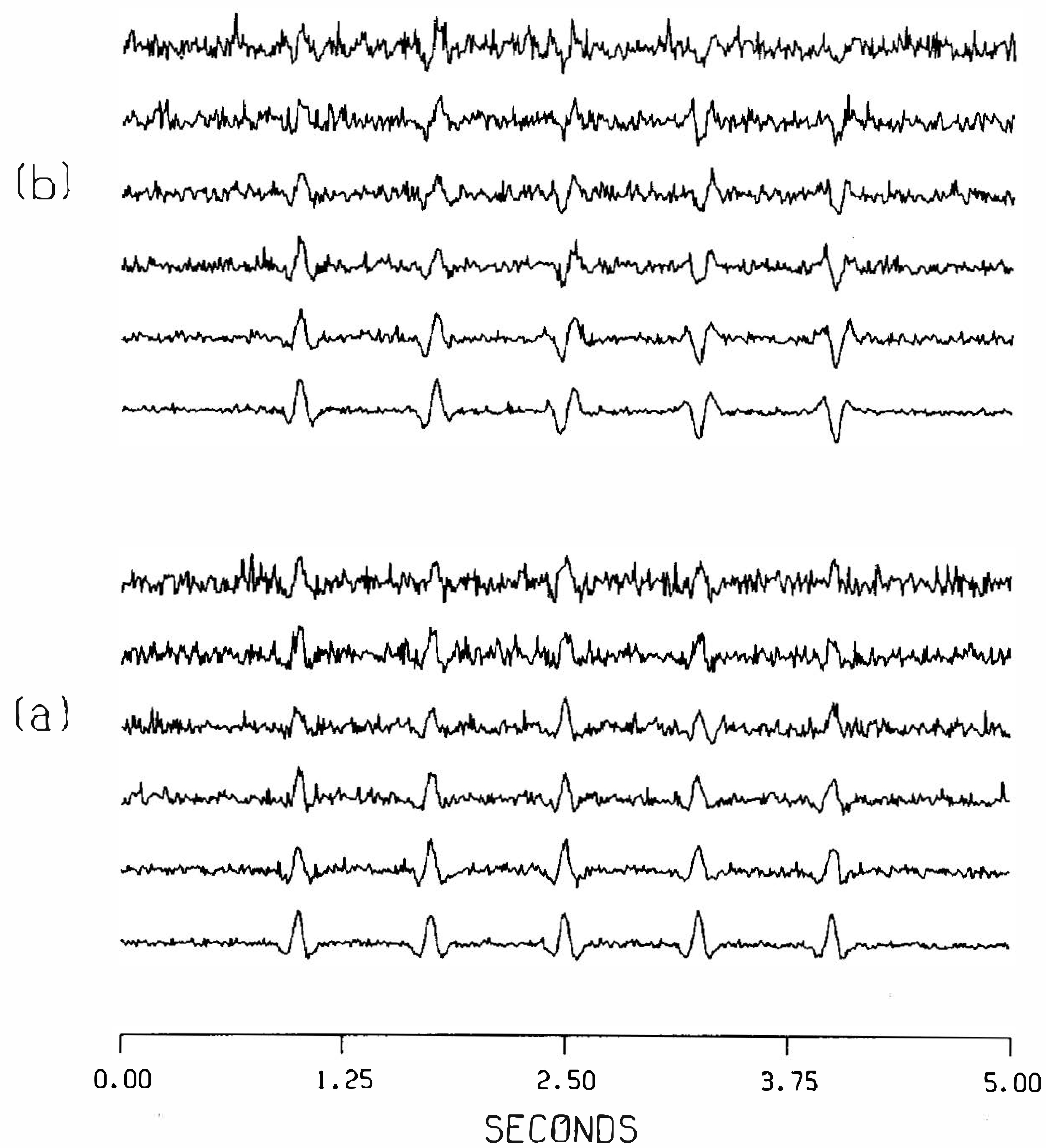

Fig. 3. Random noises added to the original Ricker wavelets. The noise level from the first (bottom) to the sixth (top)traces are 10, 20, 30, 40, 50 and 60% respectively: (a) the vertical component (b) the radial component.

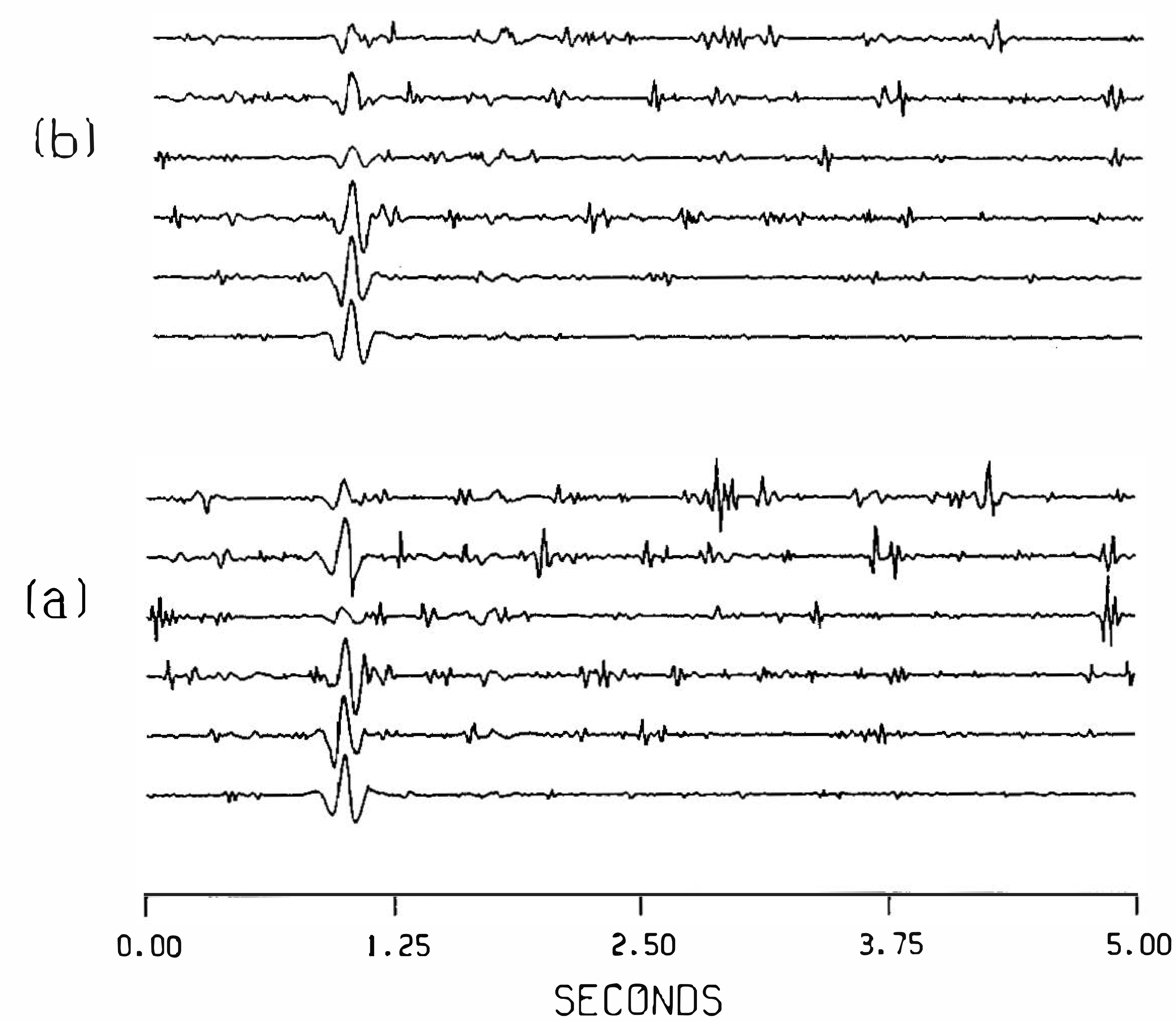

Fig. 4. Results of passing P-waves for different levels of noise added, (a) the vertical component (b) the radial component.

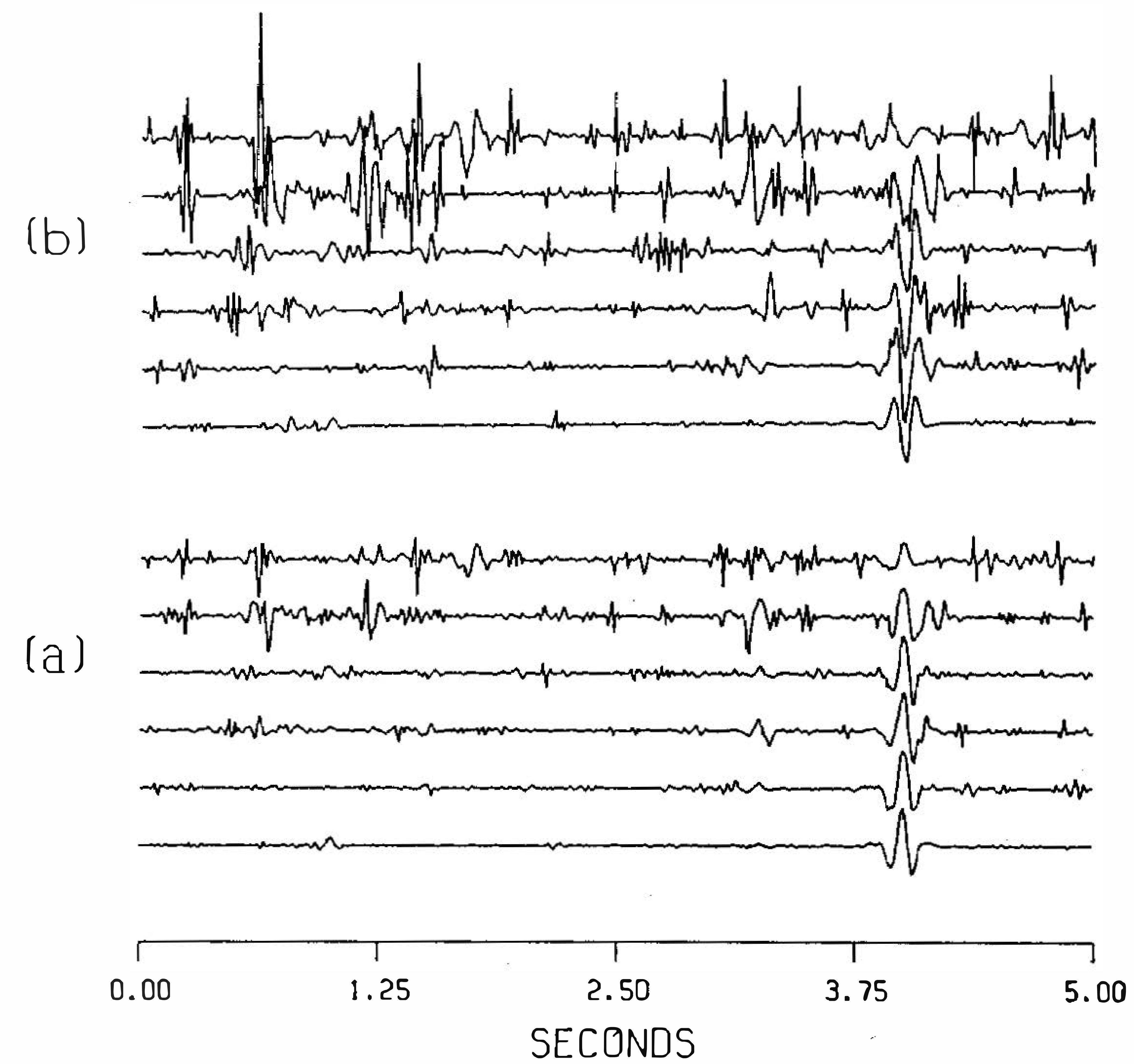

Fig. 5. Results of passing S-waves for different levels of noise added, (a) the vertical component (b) the radial component.

Acknowledgements The author appreciates the valuable suggestions by two anonymous reviewers. He also thanks the editor for his time to process the manuscripts.

REFERENCES

Flinn, E. A., 1965: Signal analysis using rectilinearity and direction of particle motion. Proc. IEEE, **53**, 1874-1876.

Jurkevics, A., 1988: Polarization analysis of three-component array data. *Bull. Seiem. Soc. Am.* **78**, 1725-1743.

Mott, H., 1986: Polarization in antennas and radar, John Wiley & Sons, 297pp.

Rene, R. M., J. L. Fitter., P. M. Forsy., D. J. Walters, and J. D. Westerman., 1986: Multi-component seismic studied using complex trace analysis. *Geophysics*, **51**, 1235-1251.

Samson, J. C., and J. V. Olson., 1981: Data-adaptive polarization filters for multichannel geophysical data. *Geophysics*, **46,** 1423-1431.

Shieh, C. F., and R. B. Herrmann., 1990: Ground roll: rejection using polarization filters. *Geophysics*, **55**, 1216-1222.

Taner, M. T., F. Koehler., and R. E. Sheriff., 1979: Complex seismic trace analysis. *Geophysics*, **44**, 1041-1063.

Vidale, J. H., 1986: Complex polarization analysis of particle motion. *Bull. Seism. Soc. Am.*, **76**, 1393-1405.