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A Study of Joint Inversion of Direct Current Resistivity, Transient Electromagnetic and Magnetotelluric Sounding Data

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ABSTRACT

Geoelectric methods have been widely used in the field of engineering and mineral exploration. Each method has its own advantage and disadvantage in interpreting depth and electrical properties of underlying subsurface. Combination of sounding data from various methods has been proved to be an effective way to improve reliability of interpretation as cited from many published literatures. Among them, several authors developed intriguing schemes to simultaneously invert two sounding data collected from two different surveying methods into a geoelectric section. These studies show that the interpreted sections based on joint-inversion scheme are superior to those from inversion of single data set alone.

This research is an extension of their work by inverting coincident loop transient electromagnetic (TEM) data, magnetotelluric (MT) data and Schlumberger sounding (DC) data simultaneously. An appropriate joint inversion scheme has been developed. Synthetic sounding data sets for DC, MT, and TEM based on a six-layer earth model were used to evaluate the performance of our scheme. Our study shows that the estimated model parameters from joint inversion of the triple data sets are more consistent with the hypothetical model than those derived by any inversion result using DC, TEM, or MT data alone. Furthermore, our result also indicates that our proposed scheme is superior to the joint inversion scheme based on DC and MT, DC and TEM, or TEM and MT.

(Key words: DC, TEM and MT, Triple joint inversion)

1. INTRODUCTION

The most common model used in the geoelectric interpretation is the horizontally layered earth. In a geoelectric survey, the depth of investigation is always limited by the topographic surface and/or available area for spread arrangement. Therefore, the direct current (DC) resistivity and transient electromagnetic (TEM) method are suitable for investigating the subsur-

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face at a depth less than 1000 m. The magnetotelluric (MT) method reveals information from deeper parts of the earth than DC or TEM method does. In addition, resolution of the layered structure in geoelectric interpretation as in general is affected by the electrical properties of the geoelectric layers. In order to overcome these shortcomings from each method, joint inversion schemes had been developed to estimate subsurface resistivity distribution. For example, Vozoff and Jupp (1975) developed an algorithm for joint inverting MT and Schlumberger sounding data. Joint inversion is also suggested by Gomez-Trevino and Edwards (1983) for controlled source EM and Schlumberger soundings, and Raiche et al. (1985) for coincident loop TEM and Schlumberger soundings. All these investigation show improvement of resolution of the interpreted layered earth structure.

To extend the work described above, we developed an algorithm of joint inversion of triple data sets of coincident loop TEM data, MT data, and Schlumberger DC sounding data. These data are inverted simultaneously in order to provide a better estimation of subsurface resistivity distribution. The algorithm has been programmed and tested by synthetic data generated from a horizontal six-layer earth model. Our results demonstrate the advantage of the triple joint inversion. Comparison made among the results from triple joint inversion, single inversion, and double inversion techniques indicates that our proposed triple inversion scheme is better in describing model parameters than the other inversion schemes. The algorithm developed by this study represents an efficient and reliable inversion approach.

2. INVERSION METHOD

The application of inversion techniques to geoelectric methods has been described before (e.g., Gomez-Trevino and Edwards, 1983; Inman, 1975; Raiche et al., 1985; Vozoff and Jupp, 1975). The Jupp-Vozoff algorithm, which is an iterative second-order Marquardt least-squares scheme, is used here. We present only the basic equations as they apply to the geoelectric problem.

Assume N unknown model parameters $\vec{x}$ are related to the M resistivity data by a nonlinear function h defined as

$$d_i = h(\vec{x}, c_i) \equiv h_i(\vec{x}); \quad i = 1, \ldots\ldots, M; \tag{1}$$

where the c_i denote the electrode separations for the DC survey, the frequencies for the MT survey, or the transient times for the TEM survey. This nonlinear equation is linearized by expansion in a Taylor series for each value of c_i with respect to a reference model x_0 . Neglecting higher order terms leads to a linear set of M equations in N unknowns,

$$\Delta\vec{d} = \underline{a}\Delta\vec{x} \quad , \tag{2}$$

where

$$\Delta\vec{d} = \vec{d} - \vec{h}(\vec{x_0}),$$

$$a_{ij} = \partial h_i / \partial x_j \Big|_{\vec{x} = \vec{x_0}}; \qquad i = 1, ...M;\ j = 1,.....N;$$

and

$$\Delta\vec{x} = \vec{x} - \vec{x_0}\ .$$

The vector $\Delta\vec{d}$ is the difference between the measured apparent resistivity, phase, or transient voltage data and the theoretical apparent resistivity, phase, or transient voltage of the initial model, respectively. The vector Δx is the difference between the unknown model parameters and the initial model parameters. In our case, the model parameter is the resistivity of each block. The matrix $\underline{a}$, referred to as the Jacobian matrix, is the matrix of partial derivatives of the data values with respect to the model parameters.

Because both the model parameters and the resistivity data values may vary over several orders of magnitude, and also because they must be constrained to positive values, we use logarithmic fitting. Thus, let **D** = ln **d**, **H** = ln **h**, and **X** = ln **x**. Using these equations and incorporating a data weighting matrix $\underline{W}$ into the Jacobian matrix and equation (2),

$$A_{ij} = W_{ij}^{1/2}(\partial H_i / \partial X_j) \tag{3}$$

and

$$\Delta\vec{D} = \underline{W}^{1/2}(\vec{D} - \vec{H}) = \underline{A}\Delta\vec{X}\ . \tag{4}$$

The inverse procedure is to find the model parameters which minimize $\left\|\Delta\vec{D}\right\|$. Since equation (4) was deduced by linearizing a nonlinear system, the inverse solution requires several iterations. At each step, equation (4) is solved for ΔX which yields a new set of model parameters. This procedure is repeated until an accepted minimum square residual is reached.

For a joint inversion of different sounding data, as shown in equation (5), the error vector $\Delta\vec{D}$ is composed of a set of vectors , ΔD

$$\begin{bmatrix} \Delta\vec{D}^{1} \\ \Delta\vec{D}^{2} \\ . \\ . \\ \Delta\vec{D}^{k} \end{bmatrix} = \begin{bmatrix} \underline{A}^{1} \\ \underline{A}^{2} \\ . \\ . \\ \underline{A}^{k} \end{bmatrix} \times \Delta\vec{X}\ , \tag{5}$$

where the elements in each subvector are the differences between the measured physical parameters and the theoretical physical parameters for a specific sounding method. The sequence of superscript shown in equation (5) represents the number of survey methods being used. Computed the model update $\Delta \vec{X}$ by equation (5), we calculate each element of the error vector, $\Delta \vec{D}^{1}$, and Jacobian matrix ΔA^{1} for different sounding surveys separately, then substitute the final results again into equation (5). Model parameters can be achieved by several iterations similar to conventional single inversion schemes.

Gloub and Reinsch (1970) decomposed the Jacobian matrix $\underline{A}$ into its row and column eigenvectors, and the associated singular values, as

$$\underline{A} = \underline{U}\underline{\Lambda}\underline{V}^{T}, \tag{6}$$

where $\underline{U}$ is an M by N data eigenvector matrix, $\underline{V}$ is an N by N solution eigenvector matrix, and $\underline{\Lambda}$ is an N by N diagonal singular value matrix.

The parameter improvement vector $\Delta \vec{X}$ is obtained by substituting $\underline{A}$ from equation (6) into equation (5). The solution is

$$\Delta \vec{X} = \underline{V}\underline{\Lambda}^{-1}\underline{U}^{T}\Delta \vec{D} \tag{7}$$

The problem is that when small singular values are present, the estimate for $\Delta \vec{X}$ is grossly contaminated by numerical noise. To overcome this problem, a damped N by N diagonal matrix $\underline{T}$ is added to equation (7); thus,

$$\Delta \vec{X} = \underline{V}\underline{T}\underline{\Lambda}^{-1}\underline{U}^{T}\Delta \vec{D} \quad . \tag{8}$$

The elements of $\underline{T}$ are

$$t_j = k_j^4 / (k_j^4 + \mu^4) \tag{9}$$

where μ is known as the relative singular value threshold and $k_j = \lambda_j / \lambda_1$. λ_j is the j th singular value and λ_1 is the maximum of the singular values. The estimate is further stabilized by initially including only the largest singular values in the estimate. As the fitting error decreases, the singular values of less importance are also included in the estimate. This is performed by initially giving μ a high value of 0.2 and then, as the fitting error decreases, permitting μ to be decreased from iteration to iteration until it reaches a minimum allowed value.

3. INITIAL DATA PREPARATION

To process efficiently Schlumberger sounding curves or Wenner field curves, Zohdy (1989) proposed a fast iterative automatic interpretation method. The algorithm is based on interpreted depths and resistivities obtained from shifted electrode spacings and adjusted apparent resistivities, respectively. The advantage of this algorithm is that it does not require an initial guess of the number of layers, their thicknesses, or their resistivities. In this paper, the iterative procedure to interpret DC field sounding curves is based on Zohdy's algorithm. Furthermore, a modified Zohdy algorithm suggested by the authors can be easily implemented to iterate the MT data, i.e., using sounding frequency shifting instead of shifting electrode spacings to obtain an initial guess of the number of layers, their thicknesses, and their resistivities for MT data inversion.

4. FORWARD MODELING

A synthetic test on noisy artificial sounding data, in which we add 5 % random Gaussian noise to the DC sounding and 10 % to the TEM and MT soundings, had been undertaken to evaluate our joint inversion algorithm. Figure1 shows the six-layer test model. The synthetic data used for inversion are the computed apparent resistivities (for DC and MT data) and transient voltages (for TEM data). Figure2 shows the Schlumberger sounding curve. Figure3 shows the computed transient voltage curve and the corresponding apparent resistivity curve of the TEM response for a coincident loop configuration. The method of Knight and Raiche (1982) was used to calculate the TEM response. The computed MT apparent resistivity sounding curve is shown in Figure 4. From these synthetic " observed " sounding data, it is obvious that electrical properties of the first to the third layer are more influential on the DC data, the third to the fifth layer on the TEM data, and the fourth to the sixth layer on the MT data.

5. INVERSION OF ARTIFICIAL DATA

Joint inversion of the DC, TEM, and MT responses by using the Jupp-Vozoff (1975) inversion scheme was carried out, a second-order Marquardt method was used to stabilize the

$\rho_1 = 200\Omega \cdot m$	$t_1 = 5m$
$\rho_2 = 800\Omega \cdot m$	$t_2 = 20m$
$\rho_3 = 80\Omega \cdot m$	$t_3 = 200m$
$\rho_4 = 200\Omega \cdot m$	$t_4 = 1000m$
$\rho_5 = 10\Omega \cdot m$	$t_5 = 500m$
$\rho_6 = 100\Omega \cdot m$	

Fig. 1. A six-layer horizontally layered model, where ρ_i is the resistivity of i-th layer and t_i is the thickness of i-th layer.

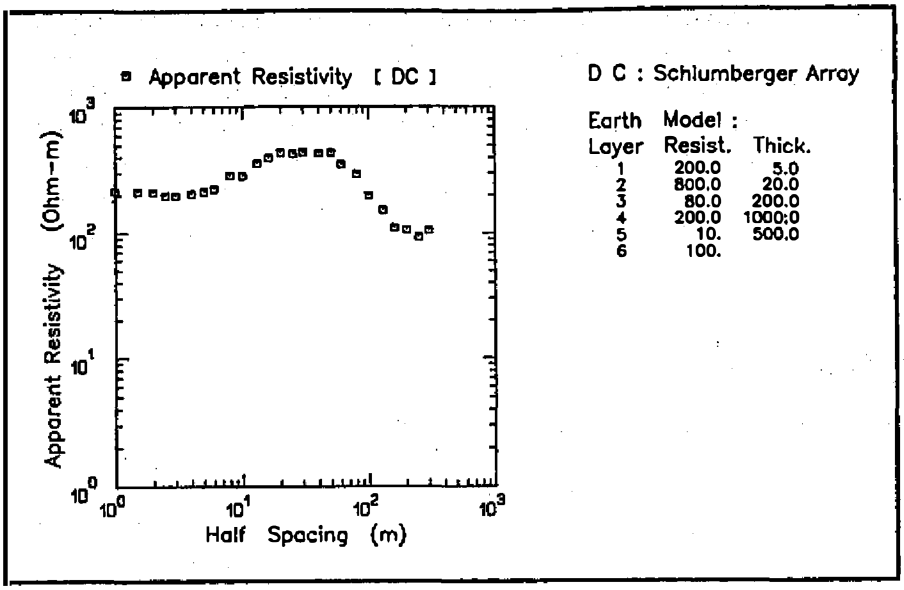

Fig. 2. Synthetic vertical electrical sounding curve with Schlumberger array. Computed data are based on the model shown in Figure 1.

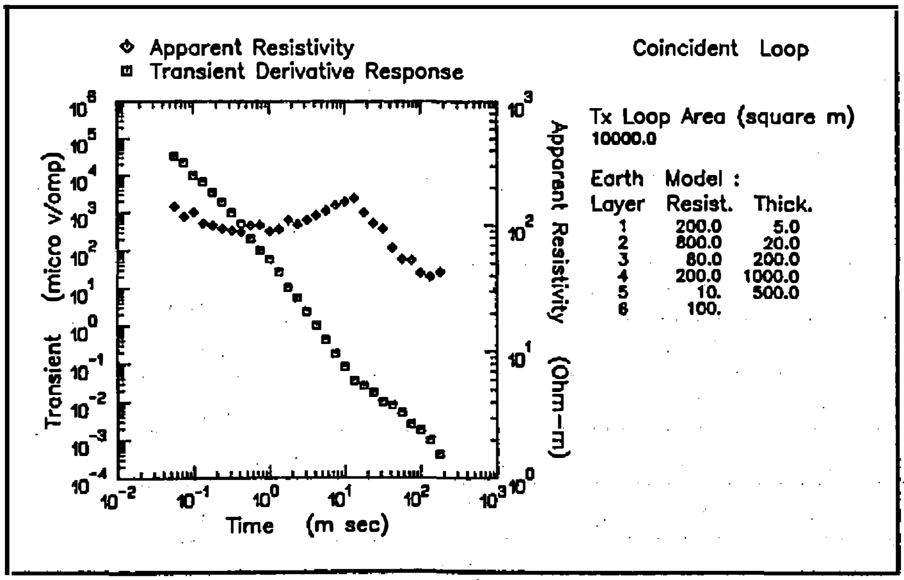

Fig. 3. Synthetic transient electromagnetic responses with a coincident loop configuration. Computed data are based on the model shown in Figure 1.

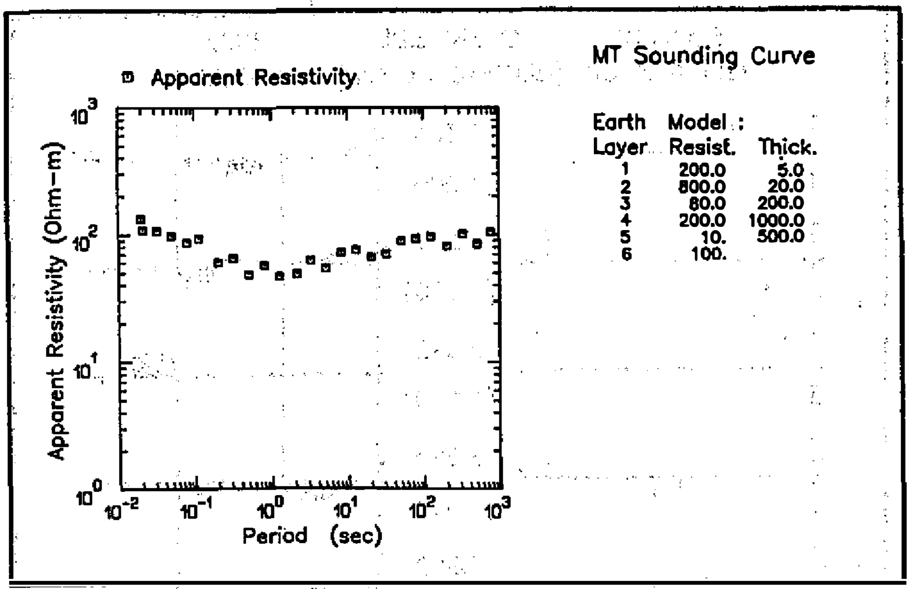

Fig. 4. Synthetic magnetotelluric apparent resistivity. Computed data are based on the model shown in Figure 1.

Table 1. Final inverted models from single DC, TEM, or MT; joint DC-MT or DC-TEM, and joint DC-TEM-MT data.

	Actual Model	Starting Model	Inversion Results					
			DC	TEM	MT	DC-MT	DC-TEM	DC-TEM-MT
ρ_1	200	400	206.8	413.3	397.4	199.1	205.9	208.6
ρ_2	800	1000	999.3	1096.2	998.9	823.1	874.6	877.2
ρ_3	80	200	79.8	82.1	135.1	92.8	82.8	80.8
ρ_4	200	500	558.8	216.3	713.7	249.4	217.0	177.1
ρ_5	10	20	20.1	14.0	13.8	7.4	13.9	10.5
ρ_6	100	300	300.9	221.1	107.2	106.3	228.1	108.0
t_1	5	2	5.5	2.2	2.0	5.1	5.2	5.2
t_2	20	10	16.6	20.6	8.5	17.9	18.9	19.7
t_3	200	500	232.4	222.8	497.7	347.2	223.9	182.3
t_4	1000	600	623.2	900.7	937.1	1153.4	897.5	1019.3
t_5	500	200	198.7	893.5	763.9	456.6	891.8	536.2

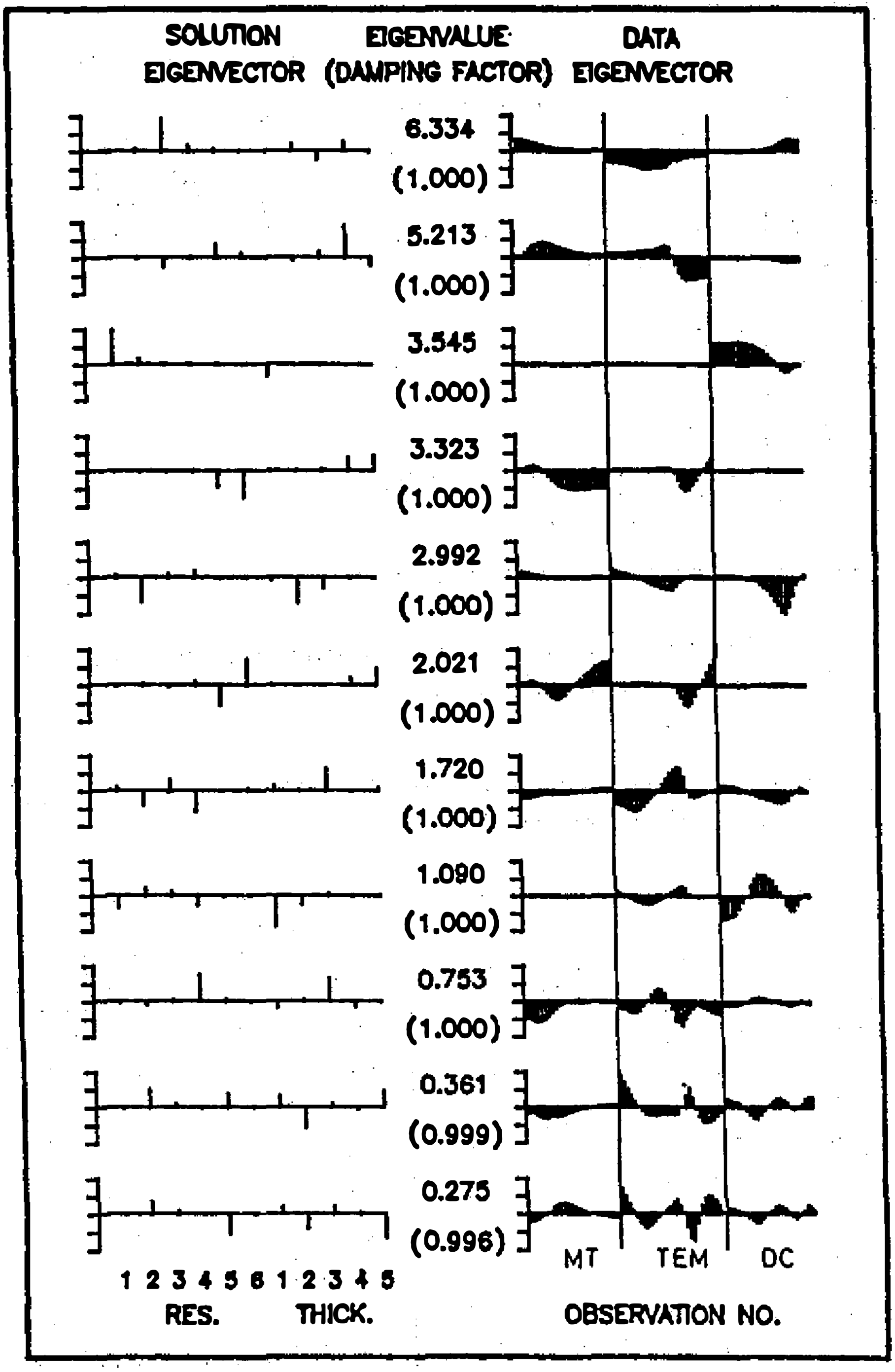

Fig. 5. Eigenvectors and corresponding eigenvalues of the model and data space for the triple joint inversion of DC, TEM, and MT data.

inversion process. The results are shown in Table 1. The results from inverting single DC, TEM; or MT; joint DC-MT; and joint DC-TEM data using the same starting model are also shown in Table 1.

Table 1 indicates that the model parameters derived from the joint inversion of DC-TEM-MT data are more consistent with the original model than those from any single or double data inversion result. Figure 5 shows the eigenvalues and the corresponding eigenvectors, eigenvalues of model and data space. The eigenvectors represent a measure of the inversion performance. The well resolved shallow layer parameters are recognized by the solution eigenvectors of the eigenvalues 3.545, 2.992, and 1.090. At the eigenvalues of 6.334, 5.213, and 1.720, it shows a good resolution of the medium deep layer parameters by the TEM sounding data. At the eigenvalues 3.323 and 2.021, the well resolved deeper layer parameters by the MT sounding data can be recognized.

6. CONCLUSIONS

In this paper an algorithm for the joint inversion of DC, TEM, and MT sounding data has been presented to solve some basic problems inherent in each geoelectric data interpretation. The shortcoming of each method may partially be compensated by the others. As pointed out by Raiche et al. (1985), resolution of layer resistivity and thickness may be achieved by using an eigenvalue analysis. Based on our model study, we have shown that the joint inversion of triple data is proved to be more effective in parameter estimation than inversion of single or inversion of double data sets.

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