

REVIEW

A Review of the Multilevel Slug Test for Characterizing Aquifer Heterogeneity

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ABSTRACT

All aquifers are heterogeneous to a certain degree. The spatial distribution of hydraulic conductivity $K(x, y, z)$, or aquifer heterogeneity, significantly influences the groundwater flow movement and associated solute transport. Of particular importance in designing an in-situ remediation plan is a knowledge of low- K layers because they are less accessible to remedial agents and form a bottleneck in remediation. The characterization of aquifer heterogeneity is essential to the solution of many practical and scientific groundwater problems. This article reviews the field technique using the multilevel slug test (MLST), which determines a series of K estimates at depths of interest in a well by making use of a double-packer system. The $K(z)$ obtained manifests the vertical variation of hydraulic conductivity in the vicinity of the test well, and the combination of $K(z)$ from different wells gives rise to a three-dimensional description of $K(x, y, z)$. The MLST response is rather sensitive to hydraulic conductivity variation; e.g., it is oscillatory for highly permeable conditions ($K > 5 \times 10^{-4} \text{ m s}^{-1}$) and a nonoscillatory for $K < 5 \times 10^{-4} \text{ m s}^{-1}$. In this article we discuss the instrumentation of the double-packer system, the implementation of the depth-specific slug test, the data analysis methods for a spectrum of response characteristics usually observed in the field, and field applications of the MLST.

Key words: Heterogeneity, Multilevel slug test, Vertical variations, Hydraulic conductivity

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1. INTRODUCTION

All aquifers are heterogeneous to a certain degree as they are composed of geological materials of different hydraulic conductivities. The spatial variability of hydraulic conductivity in an aquifer, $K(x, y, z)$, is usually referred to as aquifer heterogeneity, which significantly influences groundwater flow behavior as well as the underlying solute transport. Lack of information on $K(x, y, z)$ can lead to an incomplete understanding of groundwater movement and thus a biased interpretation of plume migration. It is well recognized that the low- K layers in a contaminated aquifer are bottlenecks to many in-situ remediation measures. This is because they are able to store massive contaminants through molecular diffusion in the contamination stage that

can last decades. In the remediation stage, the low- K layers are much more difficult to clean up because they are less accessible to the remediation agents. As a result, a back diffusion process takes place to release the contaminants stored in the low- K layers into the surrounding high- K layers which are already decontaminated to a certain degree. This back diffusion process significantly prolongs the remediation duration and lowers remediation efficiency (e.g., see Bedient et al. 1999).

Moreover, it is well recognized that the heavy breakthrough tails normally observed in either laboratory or field tracer tests are largely controlled by back diffusion, and thus the influence of the low- K layers is a dominant factor underlying the non-Fickian transport phenomena (Haggerty and Gorelick 1995; Schumer et al. 2003; Zhang et al. 2008, 2009; Bianchi et al. 2011; and references given therein).

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How to characterize aquifer heterogeneity has been a major concern in hydrogeology and contaminant hydrogeology.

1.1 An Overview of Relevant Field Technologies

There are a few different field techniques available for the characterization of aquifer heterogeneity. These techniques are generally classified as the in-well method which is conducted in wells, or the out-well method which uses no wells. The most popular out-well method is the direct-push technique. It employs a direct-push rig to drive into the aquifer the hydraulic profiling tool (HPT) which is able to continuously measure the electric conductivity distribution $E_c(z)$, and the permeameter which is used to determine K values at a few different depths (Butler et al. 2007; Dietrich et al. 2008; Liu et al. 2009; and references given therein). A correlation for $E_c(z)$ and the scattered K estimates yields a continuous distribution of $K(z)$. The in-well method utilizes appropriate hydraulic test instruments in wells. There are three different in-well techniques which are frequently used: (1) the multilevel slug test (MLST) which is implemented with the aid of a double-packer system (e.g., Zlotnik and McGuire 1998a; Zlotnik and Zurbuchen 2003; Zeman-sky and McElwee 2005; Ross and McElwee 2007; Sie 2009; Alexander et al. 2011), (2) the dipole flow test (DFT) which is conducted with the aid of a triple-packer system with a pump submerged in between two lower packers (Zlotnik et al. 1997, 2001; Zlotnik and Zurbuchen 1998; Zurbuchen et al. 1998), and (3) a borehole flow meter test (BFT) which requires no packers but a borehole flowmeter (Boman et al. 1997; Arnold and Molz 2000; Crisman et al. 2001; Paradis et al. 2011).

Each of the in-well and out-well techniques has a range of conditions and purposes for which it is best suited. For example, the direct-push technique is only applicable to unconsolidated materials with a grain size ranging from 0.02 to 4 mm. The maximum depth that the direct-push rig can reach is less than about 25 ~ 30 m due to a power limitation. Under these favorable conditions, the direct-push technique is rapid relative to in-well techniques. On the other hand, the applicability of three in-well techniques is largely affected by well conditions (e.g., well radius, well plumbness, screen length, the skin effects, and smoothness of the borehole surface or the well casing surface) instead of geological conditions. That is, they can be used in either wells in unconsolidated aquifers or open boreholes in fractured formations.

Regarding field applicability, the BFT is easiest to apply and the DFT is most difficult to use. Regarding the consistency of the test results, it was found in an alluvial aquifer that a strong correlation exists between the MLST and DFT, and a less strong correlation between the BFT and other two tests (Zlotnik and Zurbuchen 2003). Regarding the sensitivity to heterogeneity, the response of MLST has distinctive characteristics for hydraulic conductivity variations; i.e., an

oscillatory response for highly permeable conditions ($K > 5 \times 10^{-4} \text{ m s}^{-1}$) and a nonoscillatory response for moderate to low hydraulic conductivity conditions ($K < 5 \times 10^{-4} \text{ m s}^{-1}$). Such a distinction is not available in the DFT and BFT.

1.2 Scope and Purposes

This article aims to review the MLST with an emphasis on its instrumentation, and implementation as well as the evolution of the relevant models and data analysis methods. In section 2, we discuss the double packer system and its implementation with an emphasis on the test initiation which is important to consequent data quality. In section 3, typical MLST responses are discussed, and their underlying influence addressed. In section 4, the evolution of modeling the MLST in the last decade is discussed, and in section 5, two methods of analyzing oscillatory and nonoscillatory responses are given in detail for practical purposes. Some field applications of the MLST and relevant data analyses are discussed in section 6. In section 7, conclusions and suggestions are given. All the field data used in this article are from MLSTs conducted at different aquifers in Taiwan, unless otherwise noted. All the wells used are 4" in diameter ($r_w = 0.051 \text{ m}$) and the riser pipe is 1" in diameter ($r_c = 0.014 \text{ m}$).

2. INSTRUMENTATION AND IMPLEMENTATION OF MLST

A slug test determines the K value for the geological materials immediately adjacent to the test section by analyzing the test response due to an initial pressure difference w_0 between the well water and surrounding groundwater. The MLST is a series of depth-specific slug tests conducted at different depths in a well with the aid of a double-packer system. For each well, a profile of $K(z_i)$ is obtained, where z_i denotes the center depth of the test section of each slug test, and $i = 1, 2, 3, \dots, n$. For many wells, the combination of the $K(z_i)$ of each well gives a three-dimensional description of aquifer heterogeneity. We will discuss the double-packer system and the test initiation method to generate a stable w_0 , which influences the quality of the subsequent test response.

2.1 The Double-Packer System

As shown in Fig. 1, the most often used double-packer system consists of two pneumatic packers connected by a perforated stainless steel pipe (the test section) and a PVC riser pipe. The riser pipe extends the test section, through the inner channel of the top packer, to the ground surface. The length of the riser pipe can be adjusted to fit the depth of the double packer system in the well. Inside the riser pipe there is a pressure transducer submerged at a depth, D_T , to measure the pressure head change, $H(t)$, of the water col-

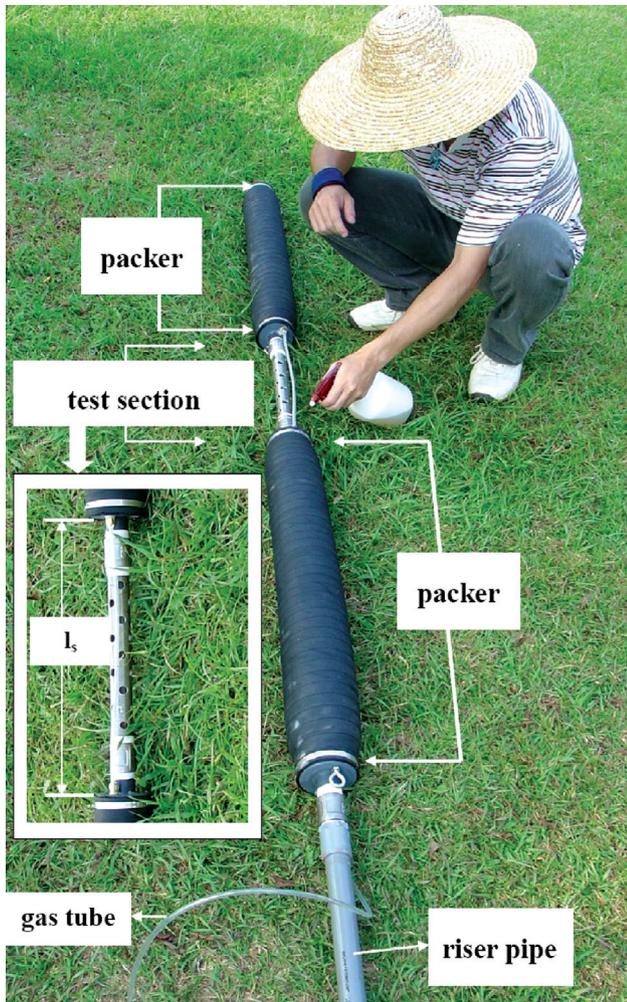


Fig. 1. The double packer system consists of two pneumatic packers connected by a test section and a riser pipe. The gas tube is used to inflate or deflate the packers.

umn above D_T . Radius of the riser pipe, r_c , therefore must be large enough to host the pressure transducer. The test section radius r_s , usually 0.0127 m (0.5 inch), has little influence on the MLST response. The test section length, l_s , however, significantly affects the resolution of the test results; that is, the vertical average effect on the estimate of K is inversely related to the test section length (Zemansky and McElwee 2005; Sie 2009). Furthermore, Sie (2009) found that the MLST responses cease to change when l_s is less than about 0.41 m in a weakly heterogeneous alluvial sandy aquifer. For highly heterogeneous conditions, this phenomenon has not been observed. A smaller l_s normally results in less vertical average effect and allows more data points of the K estimates to be collected, thereby achieving a higher resolution for $K(z_i)$. Using a large section length will not only cause an excessive vertical average effect, but also create technical difficulties in the field. For example, the total length of the double packer system alone can be easily over 3 m if l_s is longer than 1 m. Such a long double-packer sys-

tem is quite easy to break when it is being lowered down to the well. It is suggested that l_s be less than 1 m.

The pneumatic packer is made of inflatable material with a length ranging from 0.3 to 1 m, depending on the manufacturer. When deflated, the radius of the packer is smaller than the well radius, r_w , so the double-packer system can be freely lowered down to a depth of interest. Upon reaching this depth, the packers are inflated by a pressurized nitrogen cylinder at ground surface. Radius of the fully inflated packer becomes slightly greater than r_w , and thus the inflated packers are able to seal off the surrounding well screen while holding the double-packer system in place (Fig. 2). The two inflated packers effectively isolate the test section from the rest of the well, so groundwater can flow into or out of the riser pipe only through the test section. A slug test is performed and $H(t)$ measured. The temporal variation of $H(t)$ is under the influence of the hydraulic properties of the geological materials and flow conditions immediately surrounding the test section. Analyzing $H(t)$ with an appropriate method will yield an estimate of K , which represents of a vertically averaged hydraulic conductivity over the test section length. Upon completing one slug test, the packers are deflated and the double-packer system

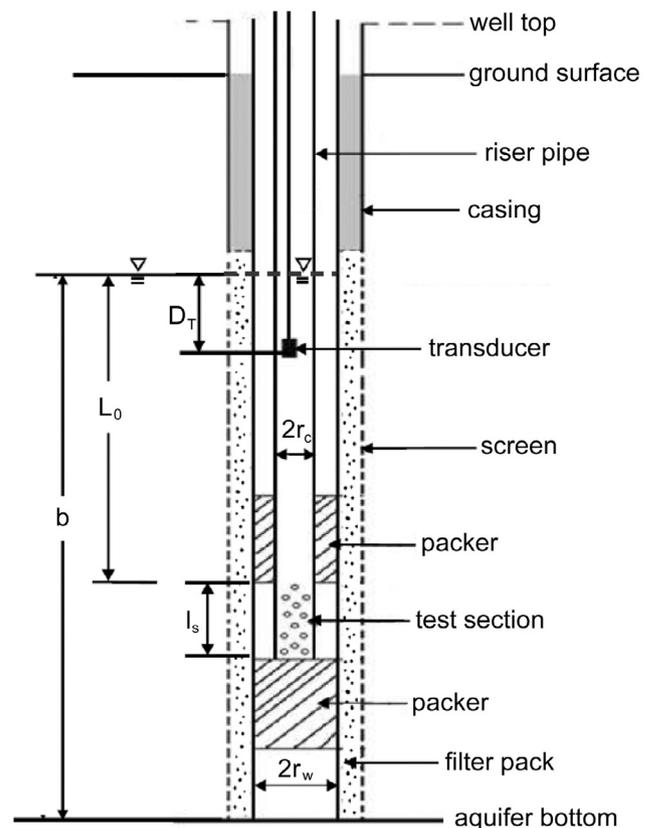


Fig. 2. Configuration of the MLST in a well. The double-packer system is fixed at a depth in the well screen section for a depth-specific slug test. The pressure transducer is placed inside the riser pipe at D_T below the initial water level.

is lowered down for a new slug test. Repeating this procedure allows us to acquire a series of $H(t)$ at different depths. A series of $K(z_i)$ are obtained from $H(t)$ of each depth.

The slug test hydraulically samples a support (or representative) volume which is considered to be a cylindrical volume of a depth equal to the screen length and a radius from the well surface into the aquifer. Beckie and Harvey (2002) found that the radius is inversely proportional to the square root of the storage coefficient. Guyonnet et al. (1993) approximate the possible radius of the support volume as $360 r_w$ for confined aquifers. For unconfined aquifers Bower and Rice (1976) gave empirical relationships approximating the radius of influence (see Appendix), which may be used to estimate the radius of the support volume for unconfined aquifers.

2.2 Test Initiation

The test initiation pertains to how to effectively create a pressure difference between the well water and surrounding groundwater. This step is important because it influences the quality of the consequential test response. Conventionally, a slug test is mechanically initiated with the dropping and removal of a solid object “slug” of some material within a well (Hvorslev 1951), or using a bailer to quickly extract or inject a certain amount of water into or from a well. These mechanical methods are difficult, and largely impractical, to create a stable initial water level change in the well, especially in highly permeable aquifers. An unstable water level makes the determination of the actual initial water level change w_0 and the associated initial water velocity $dw/dt = v_0$ very difficult. The knowledge of w_0 is necessary in data analysis of both the oscillatory and nonoscillatory responses. The initial water velocity v_0 is required as well for data analysis of an oscillatory response. Therefore, these mechanical test initiation methods are not recommended for use.

To avoid complications of an unstable initial water level, a pneumatic method was developed by Prosser (1981) which has been widely used to initiate a slug test (e.g., Butler 1998; Zemansky and McElwee 2005; Sei 2009 and others). It utilizes the same nitrogen cylinder that inflates the packers to depress the water level in the riser pipe to a displacement of w_0 . The pressure is held, usually less than 5 seconds, until the depressed water level becomes stable. This stable initial water level corresponds to an identifiable initial water level drop w_0 and a zero water velocity, $v_0 = 0$, effectively initiating the slug test with a known w_0 and v_0 . (A zero initial water velocity is more desired than a non-zero initial water velocity in the data analysis, as discussed below.) Then the injected nitrogen is released and groundwater starts to flow into the test section, resulting in a recovery of the water level in the rise pipe. The pneumatic method is best for use in highly permeable aquifers. For conditions

of moderate to low hydraulic conductivity, we found that it is as well effective (Sei 2009). To ensure that the pressure transducer is fully submerged during the test, D_T must be greater than w_0 .

3. CHARACTERISTICS OF MLST RESPONSE

As mentioned above, the MLST response is oscillatory for highly permeable conditions ($K > 5 \times 10^{-4} \text{ m s}^{-1}$) and nonoscillatory for conditions of moderate to low hydraulic conductivity ($K < 5 \times 10^{-4} \text{ m s}^{-1}$). The measurement of oscillatory response is significantly influenced by the initial water displacement w_0 and the depth of the pressure transducer, D_T .

3.1 Oscillatory and Nonoscillatory Responses

An oscillatory response is characterized by cyclic water level fluctuations around an initial water level position (Fig. 3), like medium to coarse sand. Such a highly permeable condition allows groundwater to rapidly flow into the test section. Consequently the water level in the riser pipe rises rapidly as well with significant acceleration. Due to the inertial force associated with the accelerating water movement, an uprising of the water level will not halt at the initial water level position but will pass over it until the inertial force is balanced by gravity and frictional losses. At this point the water level begins to fall, causing a fast drainage of water to the surrounding aquifer. Again due to the inertial force, the falling water level will stop at a certain depth below the initial water level position, at which the flow direc-

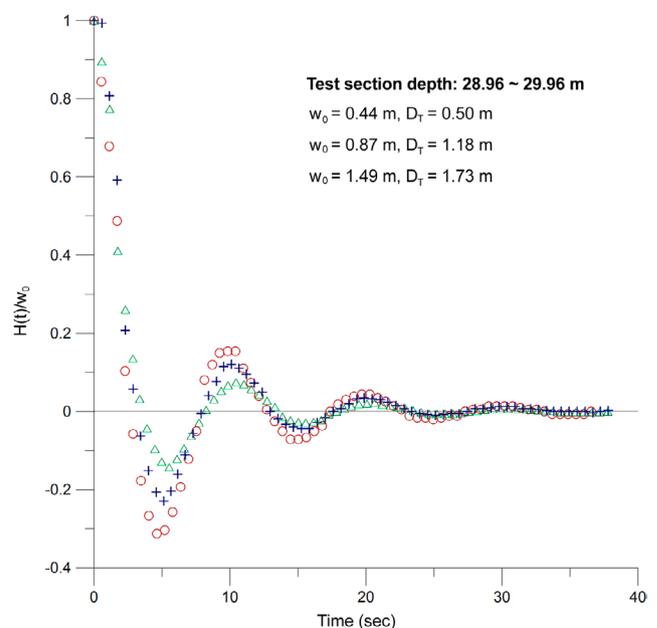


Fig. 3. Nonlinearity of $H(t)$ respect to w_0 is evidenced by the distinct normalized response of $H(t)/w_0$ for different w_0 .

tion is reversed and groundwater flows back into the riser pipe. Another rising cycle begins. After a few cycles of rise and fall, the water level will eventually stabilize at the initial position.

A nonoscillatory response usually occurs when geological materials in the neighborhood of the test section is of medium to low hydraulic conductivity (e.g., $K < 5 \times 10^{-4} \text{ m s}^{-1}$). Due to a relatively low K , groundwater slowly flows into the test section after the riser pipe is depressurized. This slow movement results in a gradual recovery of the water level from w_0 to the initial position. Variation of a nonoscillatory response usually exhibits a straight line on semilog paper, as shown in Fig. 4. The two sets of data were obtained from two MLSTs that were conducted consecutively at 29 - 30 and 30 - 31 m in the same well in a sandstone aquifer. The test section length was fixed at $l_s = 1 \text{ m}$. Visual inspection of the core samples at the two depths (Fig. 4) indicates a fairly homogeneous condition with no visible fissures or abrupt changes in geological material; the discontinuities displayed resulted from the sample retrieval procedure at the site and are not natural fractures. This apparent homogeneity is confirmed by the coincidence of the two MLST responses shown in Fig. 4.

3.2 Influence of w_0 and D_T

In a static or slowly moving body of water, hydrostatic pressure prevails and is linearly proportional to w_0 . For a non-oscillatory response, therefore, $H(t)$ measured is representative of $w(t)$ and can be directly applied to relevant models. The linearity of $w(t)$ with respect to w_0 is evidenced by the coincidence of the normalized responses of $w(t)/w_0$ of various w_0 . That is, the MLST response can be repro-

duced only when $w(t)$ is linear with respect to w_0 .

Water pressure in an accelerating water column varies nonlinearly with depth. Field experiments indicate that the oscillatory $H(t)$ is increasingly damped and shifted in phase as the transducer is submerged at a greater depth (Zurbuchen et al. 2002). Moreover, $H(t)$ measured is different from $w(t)$ (Springer 1991; Zurbuchen et al. 2002), and is nonlinearly proportional to w_0 , especially when w_0 is relatively large (Butler et al. 1996; McElwee and Zenner 1998; Zurbuchen et al. 2002). The nonlinearity of $H(t)$ with respect to w_0 is evidenced by the distinct normalized responses of $H(t)/w_0$ from three MLSTs (Fig. 3) using different w_0 and D_T ; $w_0 = 0.44 \text{ m}$ with $D_T = 0.5 \text{ m}$, $w_0 = 0.87 \text{ m}$ with $D_T = 1.18 \text{ m}$, and $w_0 = 1.49 \text{ m}$ with $D_T = 1.73 \text{ m}$. These three MLSTs were conducted in the same well in a fractured formation. The double-packer system was fixed at the depth from 28.96 - 29.96 m ($l_s = 1 \text{ m}$), where a fractured zone is identified.

The nonlinearity of $H(t)$ with respect to w_0 makes data analysis complicated, as will be discussed below. Nevertheless, this nonlinearity can be avoided when the MLST is performed following a specific operational guideline; whereby, the pressure transducer is placed about 0.5 m below the initial water level and w_0 is set less than 0.5 m (Zurbuchen et al. 2002; Butler et al. 2003). Also, at least two tests should be conducted using two different values of w_0 , both less than 0.5 m to make sure that the MLST response is linear with respect to w_0 (van der Kamp 1976; Butler 1998). This multiple test practice also serves the purpose of checking the accuracy of the MLST in terms of the data reproduction under the specified conditions. For aquifers of moderate to low hydraulic conductivity, it is also suggested that w_0 be set less than 0.5 m in order to avoid excessive recovery time of $w(t)$.

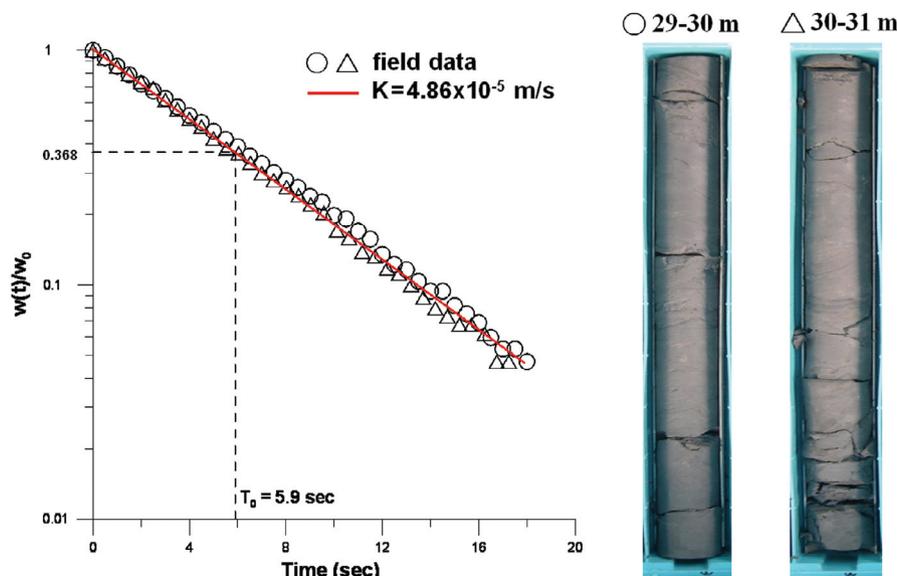


Fig. 4. Semilog plot of nonoscillatory responses from two MLSTs in a sandstone aquifer. The core samples show a relatively homogeneous condition; discontinuities are not natural fractures.

4. MODELS FOR MLST

All slug test models are formulated in terms of the water level variation in the riser pipe, $w(t)$ (Butler 1998). There are many models for a variety of slug test conditions, and we refer to Butler (1998) and Zurbuchen et al. (2002) for a comprehensive review. Herein we focus on the recent advancement of MLST models with regard to three important features, the shape factor, quasi-steady state approach, and linearity approach. Both the shape factor and the quasi-steady state approach were proposed by the first slug test model developed by Hvorslev (1951) which have been widely used in data analysis of slug tests and MLST. The nonlinear model is specifically developed to deal with an oscillatory response. Since l_s is less than an aquifer thickness in each depth-specific slug test, the partial penetration effect on the groundwater flow field in the neighborhood of the test section is taken into account in all the MLST models.

4.1 Shape Factors

The first slug test model was developed by Hvorslev (1951), in which dimensional shape factors in [L] are used to account for the influences of the test section geometry on the flow field near the test section. The shape factor, $F = Q/KH$, is determined by approximating the cylindrical well screen as a fixed spheroidal equipotential. A total of nine different shape factors were given by Hvorslev (1951) (Fig. 2) for nine different flow conditions, respectively. The shape factor for the partially penetrating condition, $F_8 = 2\pi l_s / \ln(0.5\alpha + \sqrt{1 + 0.25\alpha^2})$ with $\alpha = l_s/r_w$ the aspect ratio, has been frequently used for the MLST. Because of the simplicity, the shape factors have been widely used in modeling slug tests under a variety of hydrogeological conditions in confined aquifers. The accuracy of the shape factors was challenged by Ratnam et al. (2001). Mathias and Butler (2006) found that F_8 is incorrect. However, the error reduces to only a few percentages when α is greater than about 3.0, which is usually encountered in the field. Therefore, the Hvorslev shape factors can be used to analyze MLST response in confined aquifers.

For unconfined aquifers, Bouwer and Rice (1976) gave empirical relationships for a dimensionless shape factor, $P = Q/2\pi Kl_s H$. Note that F differs from P only by a factor of the test section length l_s which appears in the denominator of P while not in F . Recently, Zlotnik et al. (2010) employed an exact three-dimensional analytical solution to check the accuracy of the Bouwer and Rice empirical relationships. They found that the Bouwer and Rice shape factor and the exact solution are significantly different for the fully penetrating conditions, while agreeing well for partially penetrating conditions. For example, when $l_s = L_0/15$, the relative difference between the Bouwer and Rice empirical

relationship and the three-dimensional analytical solution is less than about 1 ~ 2 percent for $\alpha > 10$. For practical purposes, therefore, the Bouwer and Rice empirical relationship can be employed without introducing significant error when analyzing the MLST response in unconfined aquifers.

4.2 Quasi-Steady State Approach

The quasi-steady state approach calls for neglecting aquifer storage; e.g., the specific storage coefficient S_s in [L] can be set zero in modeling the groundwater flow induced by the test is significantly simplified. As a result, the model and hence the data analysis is significantly simplified. The appropriateness of the quasi-steady state approach has been studied by various authors. Chirlin (1989) noted that aquifer storage cannot be neglected for fully penetrating slug tests. Using the sensitivity analysis technique under various hydrogeological conditions; wherein, McElwee et al. (1995) for a homogeneous condition, Beckie and Harvey (2002) for a statistically heterogeneous system, and Audouin and Bodin (2008) for a fractured formation, it was found that the slug test response from a fully penetrating well is more sensitive to the variation of K than the variation of S_s in the sense that a non-unique combination of K and S_s can be obtained for a set of measured slug test response. For partially penetration conditions however, this is not the case and a relatively unique K can be determined without S_s ; that is, the quasi-steady state approach is valid. Hyder et al. (1994) found that aquifer storage can be neglected in low- K conditions for a broad range of the aspect ratio α . Butler and Zhan (2004) found that aquifer storage in highly permeable aquifers can be neglected for $\alpha < 130$. These criteria of neglecting aquifer storage are commonly satisfied in the field, and thus the quasi-steady state approach is reasonable for modeling the MLST responses. Of course, using the quasi-steady state model relinquishes the estimation of S_s , which is acceptable as the MLST aims to determine the $K(z)$, instead of S_s .

4.3 Nonlinearity and Linearity

The hydrodynamics of oscillating water movement in the riser pipe makes the data analysis rather complicated because of two factors. First, the relevant models are inherently nonlinear with respect to $w(t)$ (McElwee and Zenger 1998; Zurbuchen et al. 2002; Zemansky and McElwee 2005). Second, the measured $H(t)$ is not the same as $w(t)$, as discussed above. To the second point, the relationship for $H(t)$ and $w(t)$ is (Zurbuchen et al. 2002)

$$H(t) = w(t) + [D_T + w(t)]w''(t)/g \quad (1)$$

where g is the gravitational acceleration ($= 9.8 \text{ m}^2 \text{ s}^{-1}$) and

$w''(t)$ the second derivative of $w(t)$ standing for acceleration of the water level. It can be seen that $H(t)$ is quite different from $w(t)$ if the water level acceleration is significant. Based on Eq. (1) and an appropriate nonlinear model, Zurbuchen et al. (2002) develop a data analysis method, which is not easy to use. Fortunately, the complications associated with hydrodynamic pressure can be avoided by using the aforementioned operational guideline wherein w_0 is less than 0.5 m and D_T is close to the initial water level. With such a small w_0 , acceleration of the water movement is significantly reduced and pressure head at shallow depth is close to $w(t)$. As shown by Eq. (1), when both D_T and $w''(t)$ are small, $H(t)$ is approximately equal to $w(t)$. In this regard, a nonlinear model can correspondingly be linearized, and the associated data analysis becomes much simpler. It should be noted that the nonlinear model is also useful for certain cases where the oscillatory response is caused by some special well configurations; e.g., a well equipped with multiple concentric casing (Audouin and Bodin 2007) or a well with a bypass pipe (Zenner 2002).

5. DATA ANALYSIS METHODS

The methods discussed below cover a spectrum of frequently encountered nonoscillatory or oscillatory responses in terms of $w(t)$. Therefore, it is reemphasized that conduction of the MLST follows the operational guideline stated above, whereby the measured $H(t)$ is representative of $w(t)$. Each method is discussed with an application to a field example.

5.1 Data Analysis of Oscillatory Response

For analyzing oscillatory responses, an analytical solution to a linear, quasi-steady state model (Springer and Gelhar 1991) is

$$\frac{w(t)}{w_0} = \exp\left(-\frac{\beta t}{2}\right) \left[\cos(\omega t) + \frac{\beta}{2\omega} \sin(\omega t) \right] \quad (2)$$

where $\omega = 2\pi/T$ is the oscillation frequency [t^{-1}] with T the period [s], and β is the damping coefficient [t^{-1}]. Based on Eq. (2), two curve matching methods were developed by Butler et al. (2003) and Chen and Wu (2006), respectively. Both methods are not effective in application because they require matching the data points to the type curves prepared using Eq. (2) through trial-and-error procedures. To eliminate the curve matching procedures, Chen (2006) developed an analytical method that takes advantage of pertinent mathematical characteristics of Eq. (2). Later Chen (2008) proves that this analytical method is also applicable to the oscillatory responses measured by pressure transducers placed at any depth in the well. That is, the restriction of

D_T close to the initial water level set forth in the operational guideline can be relaxed when this method is used for data analysis. Ostendorf et al. (2005) also developed a closed-form solution that is applicable for pressure transducer data from any depth in a well casing. However, its application to the evaluation of K is more complicated than the method of Chen (2006). Therefore, the analytical method followed by Chen (2006) is recommended for analyzing oscillatory response from the MLST.

It is known that hydraulic conductivity K is related to β and ω through the following equation (Butler 1998; Chen 2006)

$$K = \left(\frac{\omega^2 + 0.25\beta^2}{\beta} \right) \frac{r_c^2}{r_w} F \quad (3)$$

For confined aquifers, the dimensionless shape factor based on the Hvorslev's F_8 is (Butler 1998; Chen 2006)

$$F_c = 0.5\alpha^{-1} \ln[0.5\alpha + \sqrt{1 + 0.25\alpha^2}] \quad (4)$$

For unconfined aquifers, the dimensionless shape factor is (Butler 1998; Chen 2006)

$$F_u = 0.5\alpha^{-1} \ln(R_c/r_w) \quad (5)$$

where R_c is the radius of influence that is dependent on the aspect ratio α , the test section location, and some empirical coefficients for the water table effects. The determination of $\ln(R_c/r_w)$ is based on some lengthy empirical relationships, which are given in Appendix.

Either F_c or F_u can be evaluated *a priori*. The values of β and ω ; however, need to be determined using pertinent characteristics exhibited by the MLST response. The main focus of the graphic data analysis method, therefore, is the evaluation of β and ω from $w(t)$. The damping coefficient β is related to the amplitude ratio of the oscillation (van der Kamp 1976; Chen 2006) as

$$\beta = \frac{4}{T} \ln \left| \frac{H_k}{H_{k+1}} \right| \quad (6)$$

where H_k , $k = 1, 2, 3, \dots$, is the k^{th} amplitude of oscillation occurring at t_k , $k = 1, 2, 3, \dots$. Based on Eq. (6), a graphic procedure is available for evaluating β and ω , and it is explained with the field data in Fig. 5. The MLST was conducted in a confined sandy aquifer with $l_s = 0.5$ m. The initial water level displacement is 0.45 m, and the pressure transducer was placed at $D_T = 0.5$ m. The value of F_c calculated with Eq. (4) is 0.117. The procedure of evaluating β and ω is as follows.

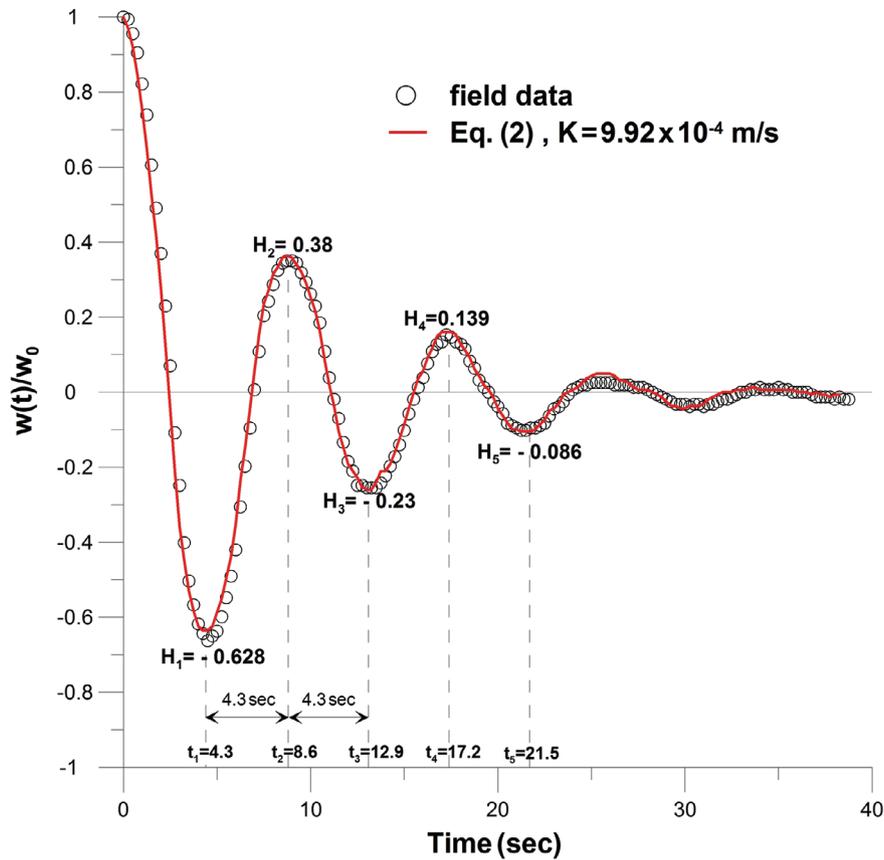


Fig. 5. Determination of the amplitude ratio and period for oscillatory response.

1. Plot the normalized response $w(t)/w_0$ versus time, as shown in Fig. 5.
2. Determine t_k of the discernable minimum or maximum points shown on the data plot. There are five such extreme points occurring at $t_1 = 4.3$ s, $t_2 = 8.6$ s, $t_3 = 12.9$ s, $t_4 = 17.2$ s and $t_5 = 21.5$ s. The period is determined as $T = t_{k+2} - t_k = 8.6$ s, so the frequency is calculated as $\omega = 2\pi/T = 0.73$ s⁻¹. It should be noted that the maximum at $t = 0$ pertains to the initial water level displacement and is not considered as the first amplitude.
3. Determine the (normalized) amplitude H_k of each t_k : $H_1 = -0.628$, $H_2 = 0.38$, $H_3 = -0.23$, $H_4 = 0.139$ and $H_5 = -0.086$. Apply all pairs of H_k and H_{k+1} to Eq. (6) to evaluate β . In theory the amplitude ratio $|H_k/H_{k+1}|$ is constant. However, the visual error in identifying H_k and t_k as well as the measurement error of $H(t)$ will result in different values for $|H_k/H_{k+1}|$. The average of the β values obtained is taken as the estimate of β . For the field data, the four amplitude ratios are 1.653, 1.652, 1.655, and 1.616, which respectively corresponds to $\beta = 0.234$, 0.233, 0.234, and 0.223 s⁻¹. So the average of the four β values, 0.231 s⁻¹, is taken as the estimate of β .
4. Use Eq. (3) to determine K with $\omega = 0.73$ s⁻¹, $\beta = 0.231$ s⁻¹ and $F_c = 0.117$. The estimate of K is 9.92×10^{-4} m s⁻¹.

5.2 Nonoscillatory Response

The first slug test model was given by Hvorslev (1951) for low- K conditions in confined aquifers

$$\ln \left[\frac{w(t)}{w_0} \right] = - \frac{Kr_w}{Fr_c^2} t \quad (7)$$

The data analysis of nonoscillatory response is based on Eq. (7), by which it is known that the normalized response on semilog paper will exhibit a declining straight line of a slope equal to $-(Kr_w/Fr_c^2)$. According to this characteristic, the first step of the data analysis is to plot the measured $w(t)/w_0$ on semilog paper, as shown in Fig. 4. Then a straight line is plotted to fit most data points, and denote the time corresponding to $w(t)/w_0 = 0.368$ as T_0 ; $T_0 = 5.9$ s in Fig. 4. It is straightforward enough to prove that T_0 is equal to the inverse of the slope of the fitted straight line, by noting $\ln(0.368) = -1$. As a result, K is determined as

$$K = \frac{F_c}{T_0} \frac{r_c^2}{r_w} \quad (8)$$

Accordingly, K is calculated to be 4.86×10^{-5} m s⁻¹, using $r_w = 0.051$ m, $r_c = 0.014$ m, $F_c = 0.075$, and $T_0 = 5.9$ s.

Although the storage effect is largely negligible for $l_s < 130 r_w$, still it may impose an influence on the MLST response. When the storage effect is in effect, the MLST response is no longer a straight line but a concave downward curve, as shown in Fig. 6 where the MLST was conducted in a mudstone aquifer. Such a nonlinear characteristic is generally not observed under permeable conditions while it appears in aquifers of relatively hydraulic conductivity, as the storage effect is generally larger in fine materials than in coarse materials (Freeze and Cherry 1979). Because the aquifer storage effect is not negligible, the data analysis requires the use of a transient model, of which no simple graphic technique is available. A three-dimensional transient model, similar to the KGS model presented by Hyder et al. (1994), is employed with an automated least square regression algorithm to estimate K and S_s , the specific storage coefficient. The estimates obtained are $K = 1.48 \times 10^{-7} \text{ m s}^{-1}$ and $S_s = 2.24 \times 10^{-6} \text{ m}^{-1}$. It is seen that the model prediction fits the measured data very well, confirming the existence of the storage effect.

Concerning unconfined aquifers, a two-section characteristic for the MLST response (Fig. 7) may appear when the well is partially submerged with a thick filter pack and the aquifer is of relatively low hydraulic conductivity. "Partial submerged" is referred to as the condition where the water table is lower than the top of the well screen, so only the lower portion of the well is below the water table. The first section is much steeper than the second, reflecting a rapid drainage of the filter packer followed by a much slower response controlled by the hydraulic conductivity of the native aquifer (Bouwer 1989). Therefore, the data analysis excludes the use of the data points belonging to the first section, while drainage from the filter pack is taken into account by making use of an effective radius, r_e . A modified

graphic technique was proposed by Bouwer (1989) to determine r_e and K . First a straight line is plotted passing through most data points of the second section, of which T_0 is determined; $T_0 = 8 \text{ s}$, as indicated in Fig. 7. The test section length is $l_s = 0.25 \text{ m}$ and then the fitted line is extended to intersect the ordinate at w^* ($= 0.56$), whereby the effective radius is calculated as $r_e = r_w / \sqrt{w^*} = 0.068 \text{ m}$. This r_e is used in Eq. (8) to replace r_w for the determination of K . Using the relationships in Appendix, F_0 is determined as 0.135. Then K is calculated to be $4.85 \times 10^{-5} \text{ m s}^{-1}$.

6. A FIELD EXAMPLE FOR HIGH-RESOLUTION $K(z)$

We now discuss a field example which shows a high-resolution $K(z)$ determined with a series of MLSTs conducted in a 4" well in a confined alluvial sandy aquifer. The aquifer consists of a silty layer from the ground surface to about 9 m, and a sandy layer from 9 m to about 18.5 m. Below 18.5 m to at least 20 m exists the bedrock. The well casing extends through the silt layer, while the well screen spans in the sandy layer. The water level was about 3.95 m below the ground surface, and thus the sandy layer is under a confined condition. A total of 28 MLSTs were accomplished at consecutive depths over the full screen length, 9.43 m using $l_s = 0.41 \text{ m}$. The operational guideline stated above was applied to each MLST. At a few depths, two different w_0 , both less than 0.5 m, were used to check the linearity of the responses.

Out of the 28 tests, there are 20 oscillatory responses and 8 nonoscillatory responses of the straight line characteristic. The estimates of K of the 20 oscillatory responses exceed $5 \times 10^{-4} \text{ m s}^{-1}$, while the estimates of K of the 8 nonoscillatory responses are less than $2 \times 10^{-4} \text{ m s}^{-1}$. As shown

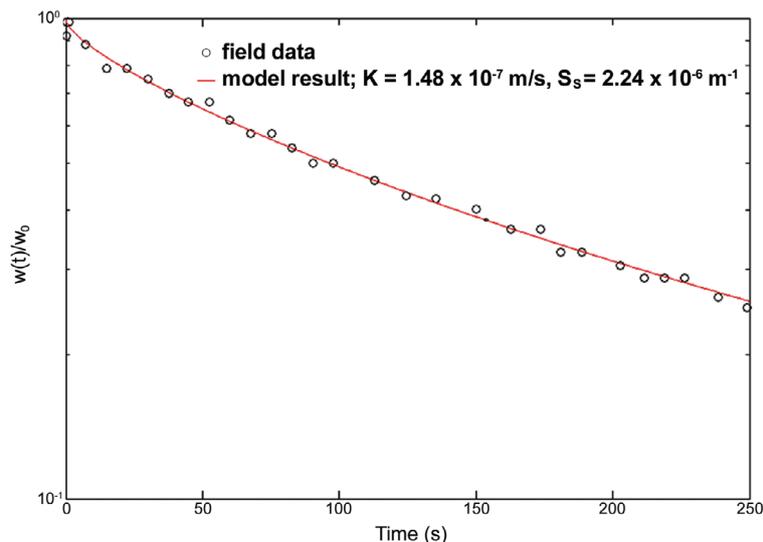


Fig. 6. The aquifer storage effect results in a concave downward trend for nonoscillatory response on semilog plot.

in Fig. 8, the vertical distribution of $K(z)$ varies with depth in an irregular way which corresponds well with the core sample description available to us, except for the depth from about 12 to 12.5 m. The core sample description of this section is sandy gravel, while the two associated K values fall in the category of much finer, less permeable materials like silty sand. This inconsistency may be due to overlooking this relatively thin layer when classifying the core samples. Missing a thin layer of fine materials may cause significant misinterpretation of subsurface DNAPL movement (Bedient et al. 1999). DNAPL is the acronym for the “dense non-aqueous phase liquid,” and chlorinated solvents are commonly discovered DNAPL at electronics manufacturing.

7. CONCLUSIONS AND SUGGESTIONS

Aquifer heterogeneity $K(x, y, z)$ significantly influences the groundwater movement and the associated solute transport. The characterization of aquifer heterogeneity is essential to the solutions of many practical and scientific

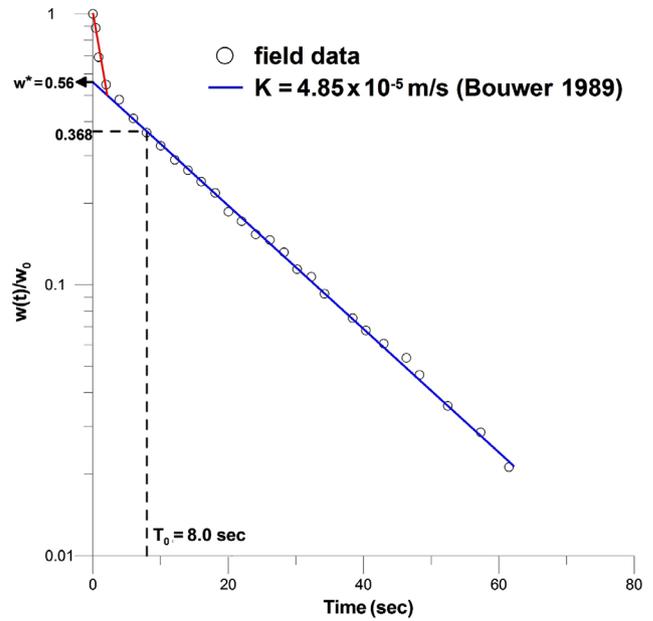


Fig. 7. Fast drainage from the filter pack results in a two-sectioned trend for nonoscillatory response in an unconfined aquifer.

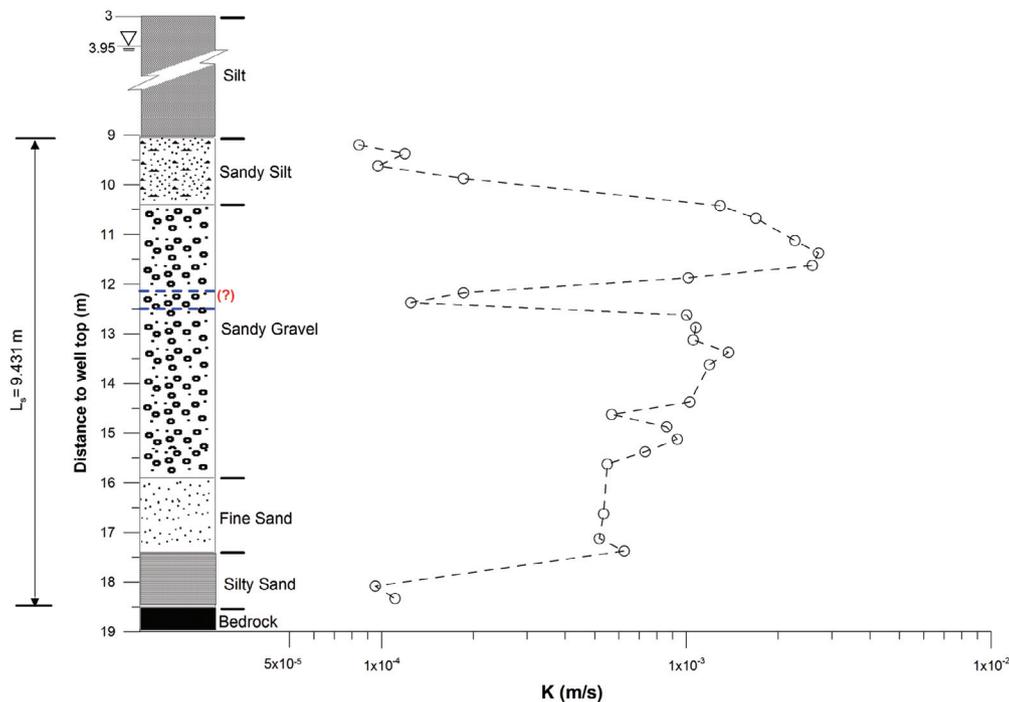


Fig. 8. A high-resolution $K(z)$ acquired using a series of MLSTs with $l_s = 0.41$ m in a sandy aquifer.

groundwater problems. The multilevel slug test (MLST) is a useful technique, which determines a series of K estimates at depths of interest in a well by making use of a double-packer system. The $K(z)$ obtained manifests a vertical variation of hydraulic conductivity in the vicinity of the test well, and the combination of $K(z)$ from different wells gives rise to a three-dimensional description of $K(x, y, z)$.

The MLST response is rather sensitive to hydraulic conductivity variation. For example, it is oscillatory for highly permeable conditions ($K > 5 \times 10^{-4} \text{ m s}^{-1}$) while nonoscillatory for conditions of moderate to low hydraulic conductivity ($K < 5 \times 10^{-4} \text{ m s}^{-1}$). For an oscillatory response, the head measured is not necessarily the same as the water level variation, and the associated data analysis is rather complicated.

To avoid such complication, an operational guideline for the MLST is recommended; that is, the pressure transducer is placed at about 0.5 m below the initial water level and the initial water displacement is set less than 0.5 m. Under such a condition, the head measured is representative of the water level change and simple data analysis methods can be used. Some pertinent graphic data analysis methods are available and discussed for a spectrum of MLST responses frequently observed in confined or unconfined aquifers. Field examples indicate that the MLST is able to determine hydraulic conductivity of different depths and yield a high-resolution $K(z)$ when a short test section length is used.

The MLST (and any other single-well hydraulic tests) is subject to the skin effect associated with the “damaged zone” in the close vicinity of the well. When the damaged zone is caused by drilling mud invasion, the hydraulic conductivity surrounding the well is reduced and the skin effect is called positive skin. When the damaged zone is created by overdevelopment of the well, the hydraulic conductivity surrounding the well is increased and the skin effect is called negative skin. For low- K conditions in either confined or unconfined aquifers, it is known that positive skin tends to cause an underestimation of K while negative skin causes an overestimation (Hyder et al. 1994; Hyder and Butler 1995; Yang and Gates 1997). For high- K conditions, the skin effect on the MLST is under continuing examination. Thus far, there is no method able to correct the error introduced by the skin effect to the estimate of K . Before such a method is available, it is suggested that either the well be thoroughly developed before the test in order to minimize the possible a skin effect, or no drilling mud is allowed to use while installing the well. and that using no drilling mud in installing the wells for hydraulic tests.

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APPENDIX

The following empirical relationships (Butler 1998) are used to calculate F_u in Eq. (5). As shown in Fig. 2, L_0 is the water depth above the top packer; other symbols are defined in the text.

$$\ln\left(\frac{R_e}{r_w}\right) = \left[\frac{1.1}{\ln(L_0/r_w + \alpha)} + \frac{A}{\alpha} + \frac{B}{\alpha} \ln\left(\frac{b-L_0}{r_w} - \alpha\right) \right]^{-1} \quad (A1)$$

$$A = 1.472 + 3.537 \times 10^{-2} \alpha - 8.148 \times 10^{-5} \alpha^2 + 1.028 \times 10^{-7} \alpha^3 - 6.484 \times 10^{-11} \alpha^4 + 1.573 \times 10^{-14} \alpha^5 \quad (A2)$$

$$B = 0.237 + 5.151 \times 10^{-3} \alpha - 2.682 \times 10^{-6} \alpha^2 - 3.491 \times 10^{-10} \alpha^3 + 4.738 \times 10^{-13} \alpha^4 \quad (A3)$$